

# Wormhole

A Fast Ordered Index for In-memory Data Management

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# Ablauf

- Intro
- O-Notation
- Hashtabelle
- B+ Baum
- Prefix Baum
- Evaluation
- Zusammenfassung
- Quellen

# Motivation

- Große Datenmengen
- Viel Memory
- Wenig Zeit
- Key-value Store
- Range query

# O-Notation

- $O: \leq$
- $o: <$
- $\Omega: \geq$
- $\omega: >$

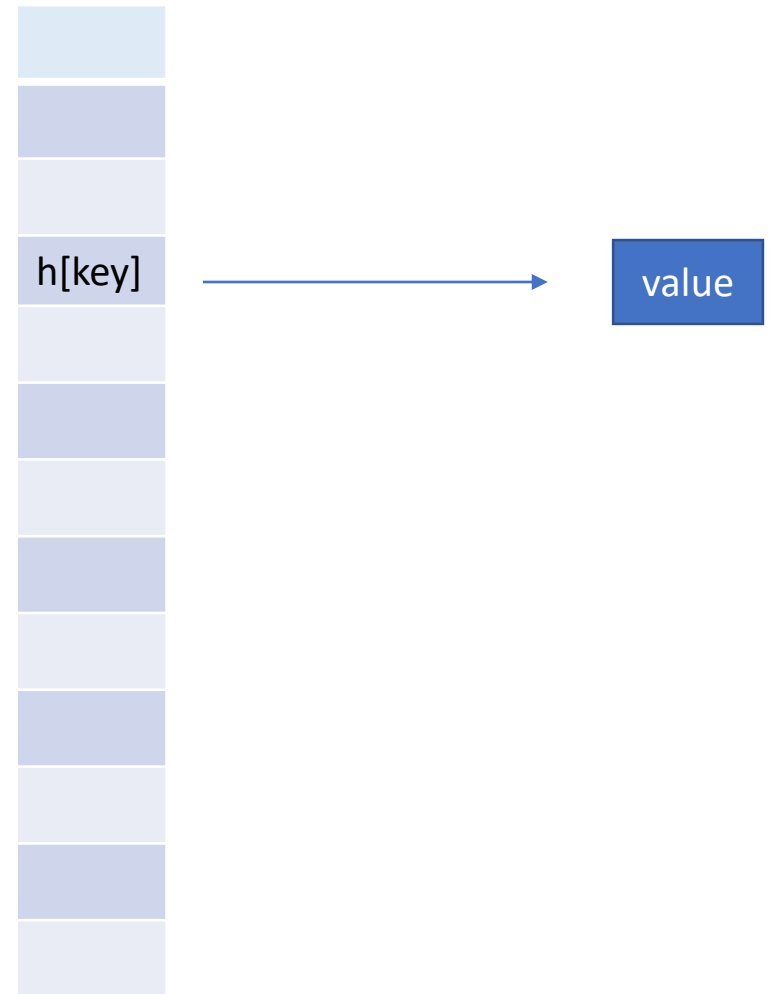
# O-Notation

Bsp:

```
1
2 for i=0, i<n, i++:
3     do stuff
4
5
6 for i=0, i<500000000, i++:
7     do stuff
8
9
10 for i=1, i<n, i**2:
11     do stuff
12
13
14
15
16
17
18
```

# Hashtabelle

- Key Value Paare
- Berechne Index aus  $h[\text{key}]$
- SET, DEL und SEARCH in  $O(1)$
- Kollisionsbehandlung

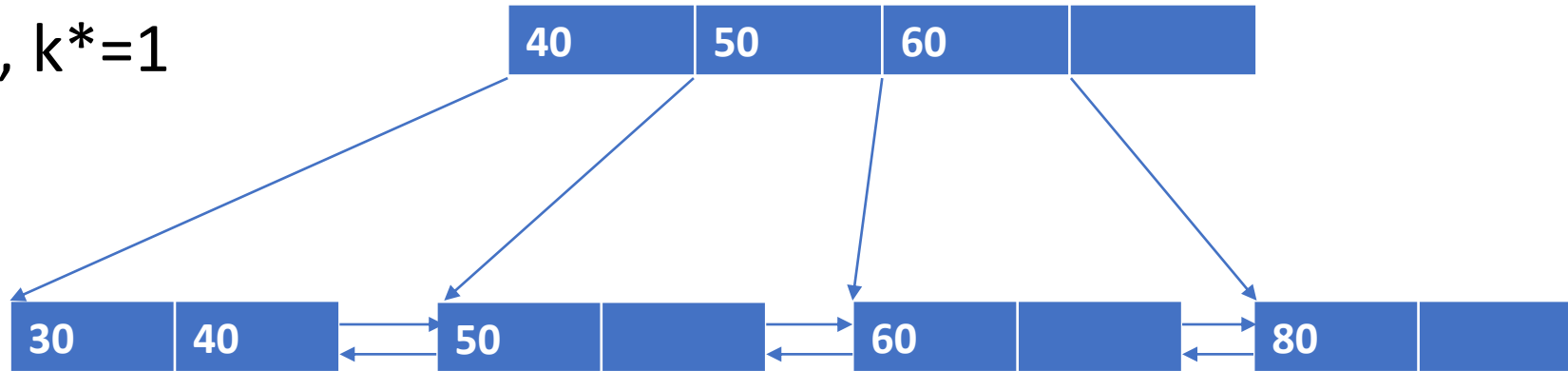


# B+ Baum

- $h^* \geq 0, k, k^* > 0$
- Jeder Pfad von Wurzel zu Blatt ist  $h^*-1$  lang
- Jeder innere Knoten hat mind  $k+1$  Kinder
- Jeder innere Knoten hat höchstens  $2k+1$  Kinder
- Jedes Blatt hat mind  $k^*$  und höchstens  $2k^*$  Einträge
  
- $O(\log N)$  lookup

# B+ Baum

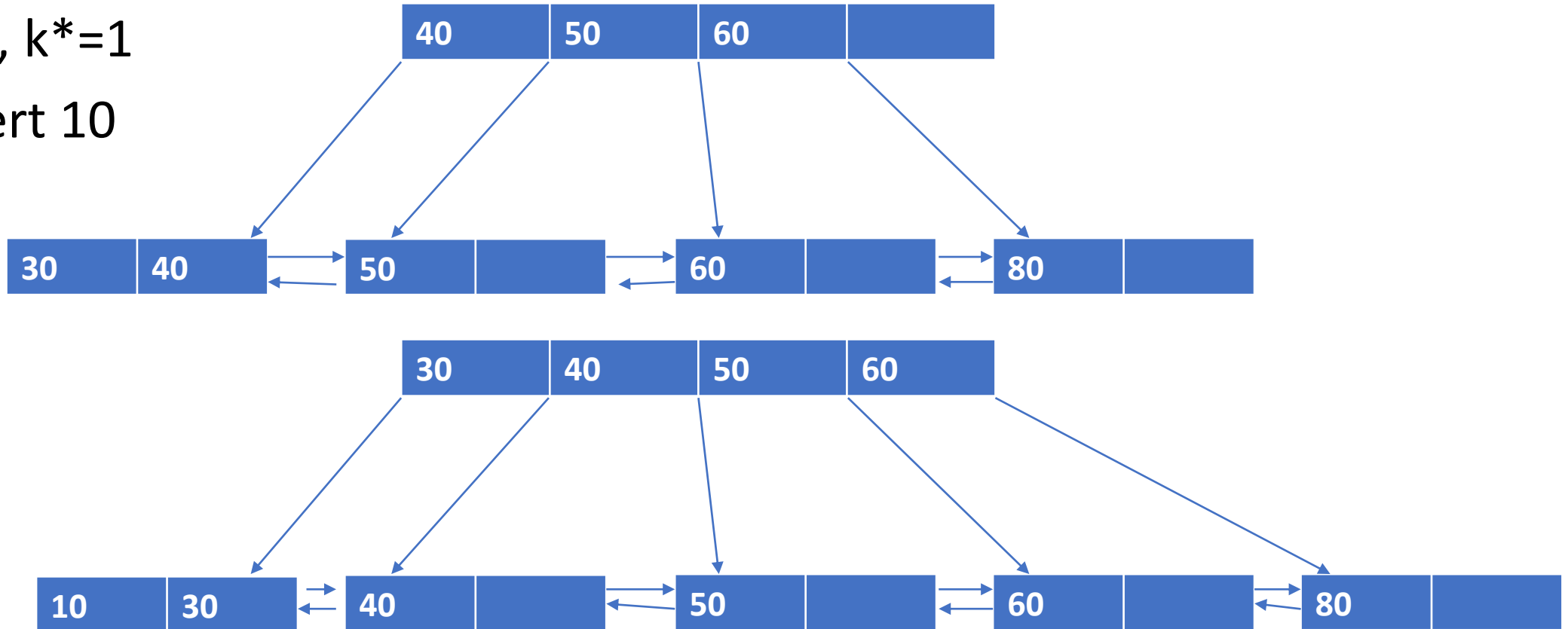
- $k=2, k^*=1$





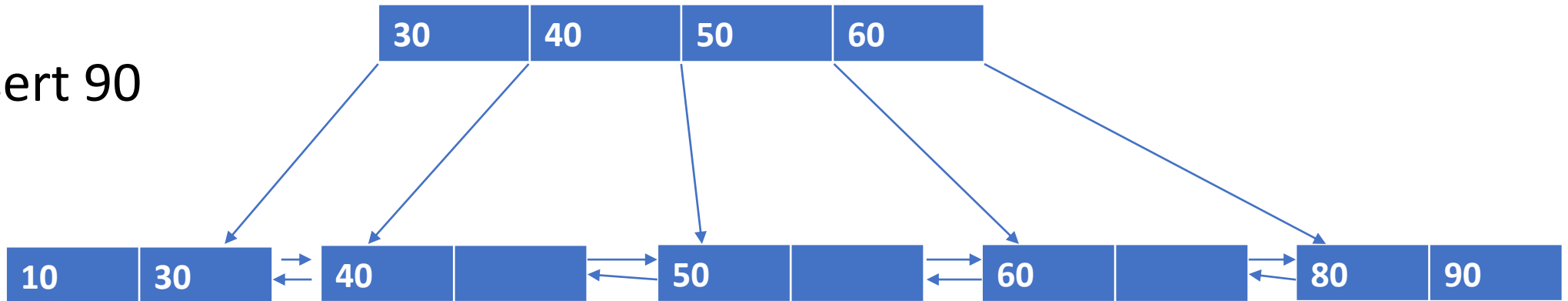
# B+ Baum

- $k=2, k^*=1$
- Insert 10

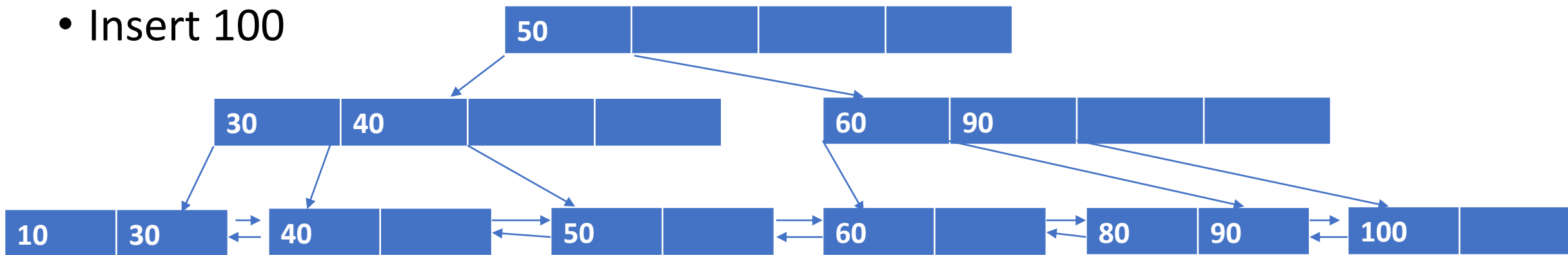


# B+ Baum

- Insert 90



- Insert 100



# Prefix Baum

- Auch Trie genannt
- Baum aus Strings
- $O(L)$  wobei  $L$  Länge der Strings ist
- Höhere Platzkomplexität
- Auto-complete

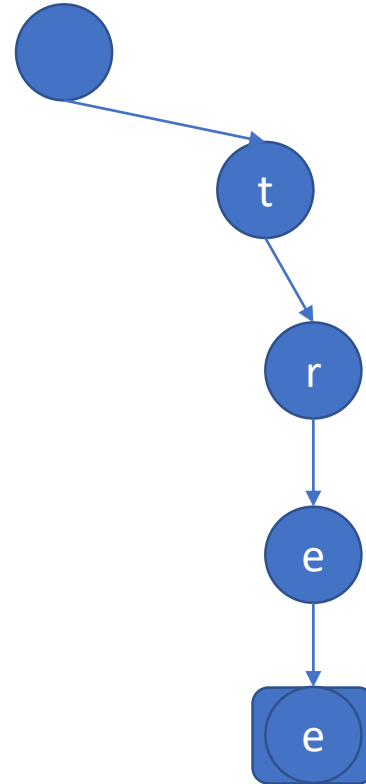
# Prefix Baum



- Insert `tree`

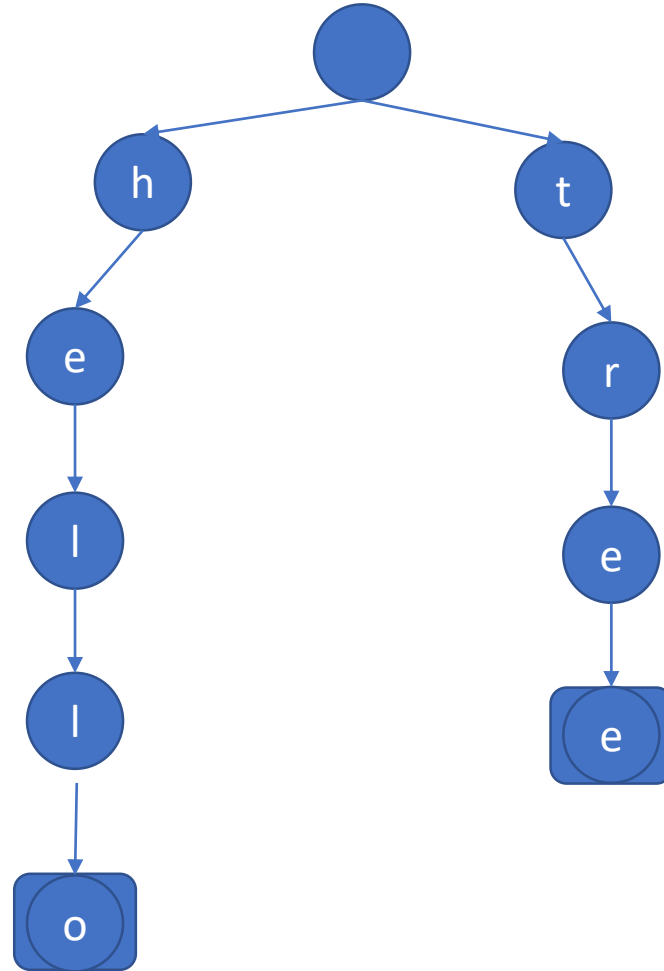
# Prefix Baum

- Insert `tree`



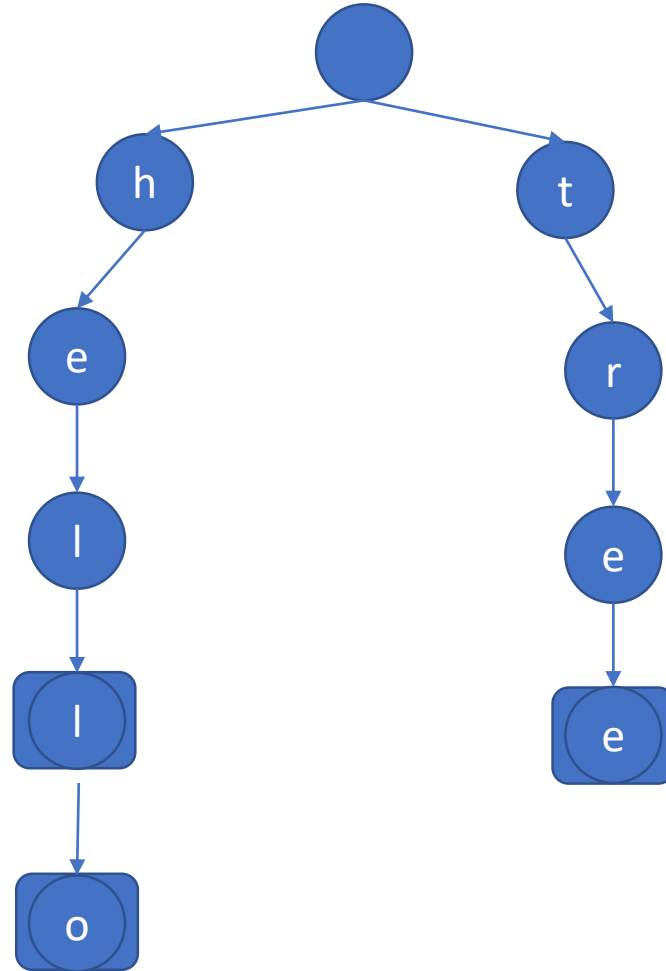
# Prefix Baum

- Insert `hello`



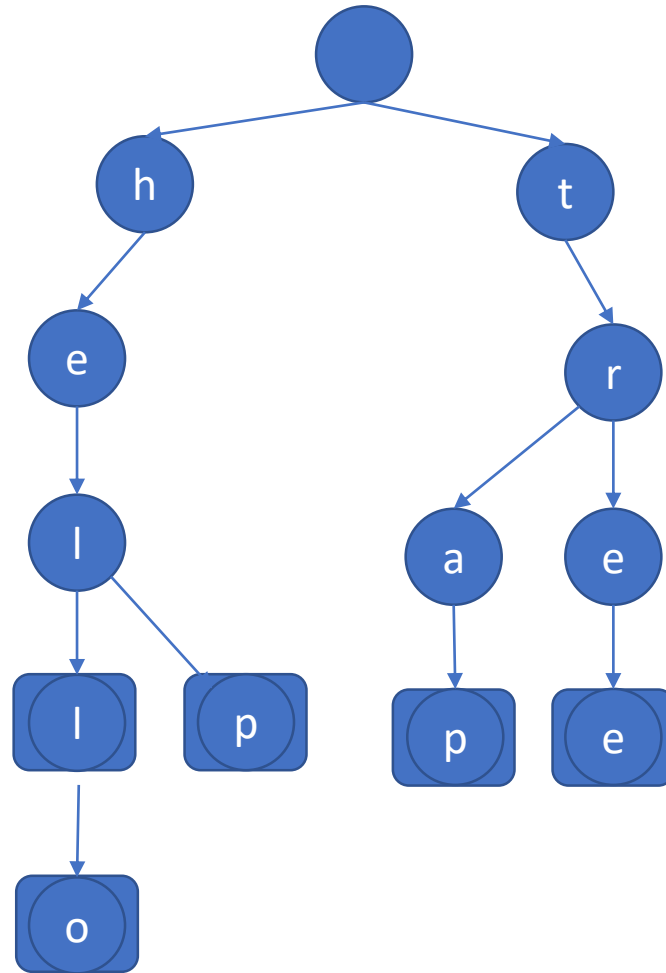
# Prefix Baum

- Insert `hell`



# Prefix Baum

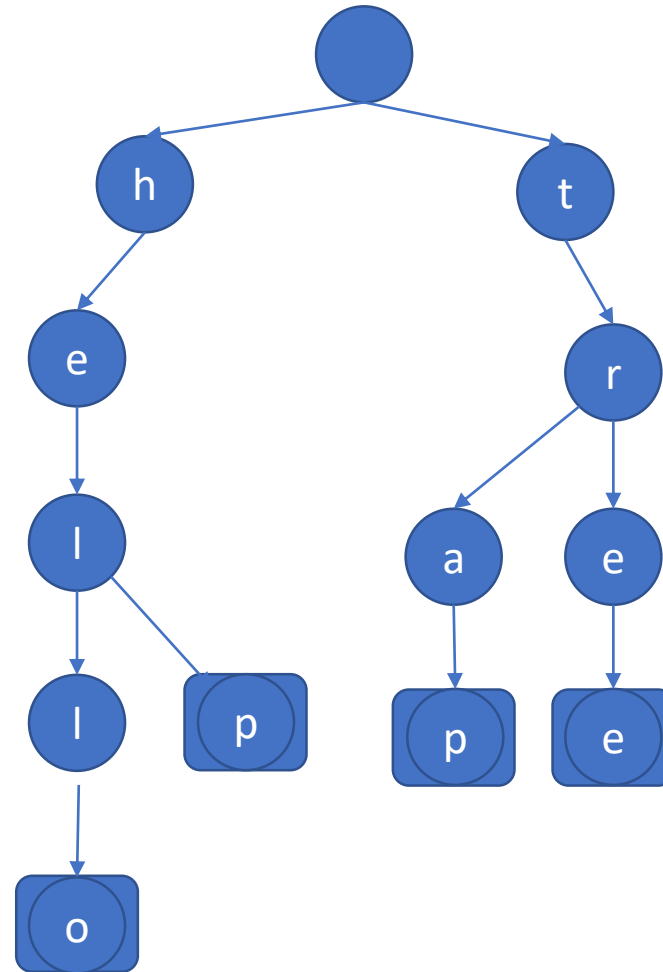
- Insert `help`
- Insert `trap`





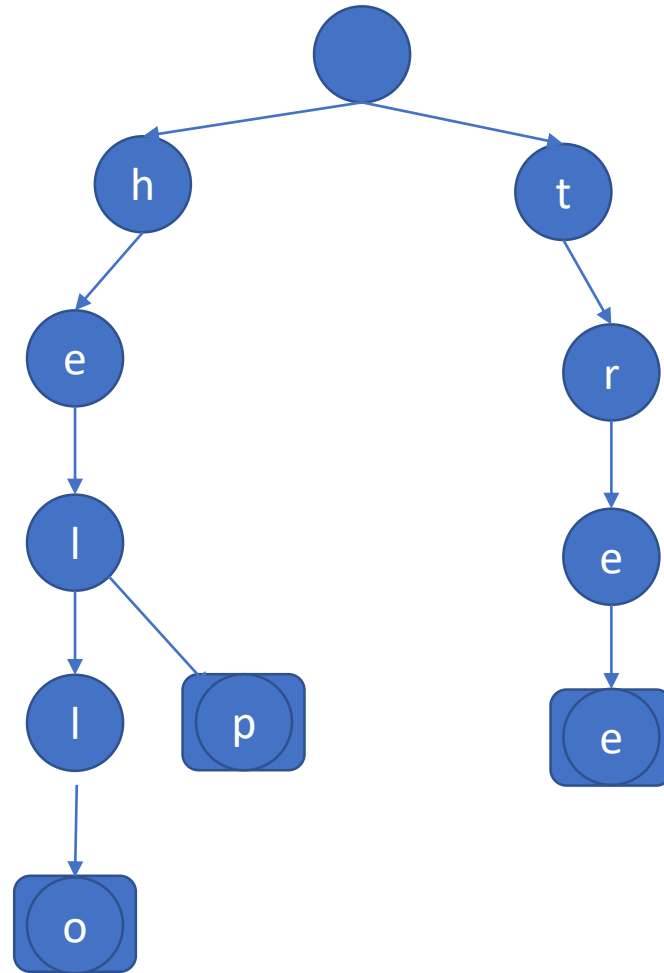
# Prefix Baum

- Delete `hell`

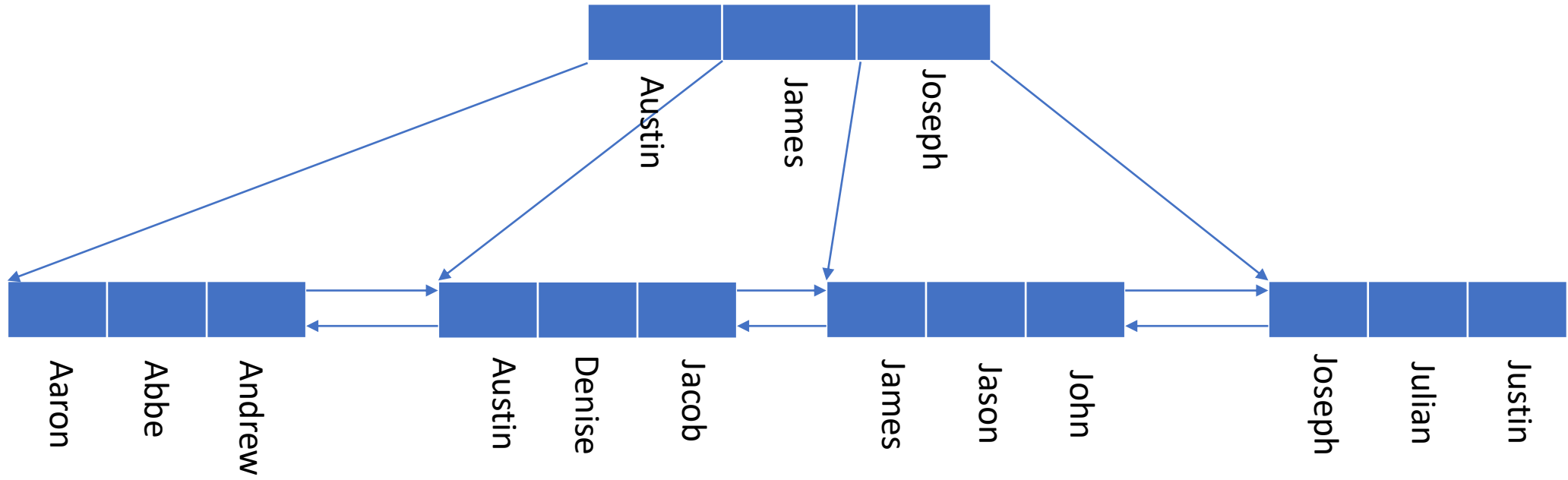


# Prefix Baum

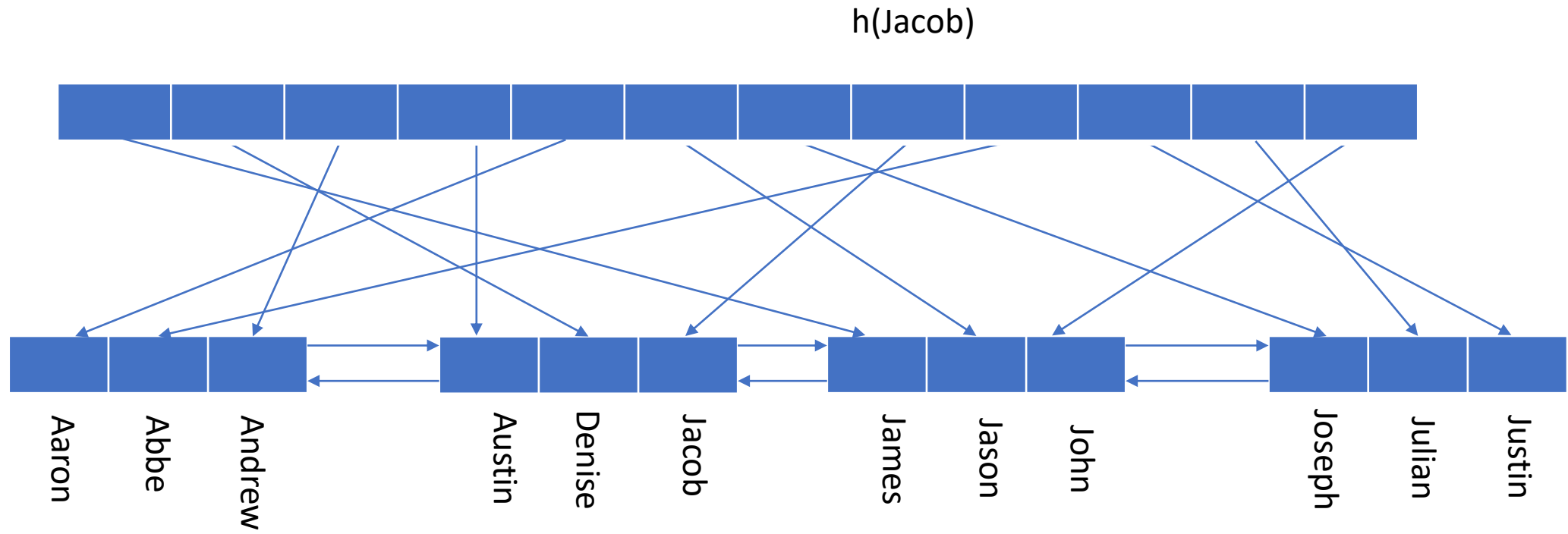
- Delete `trap`



# Wormhole [1]

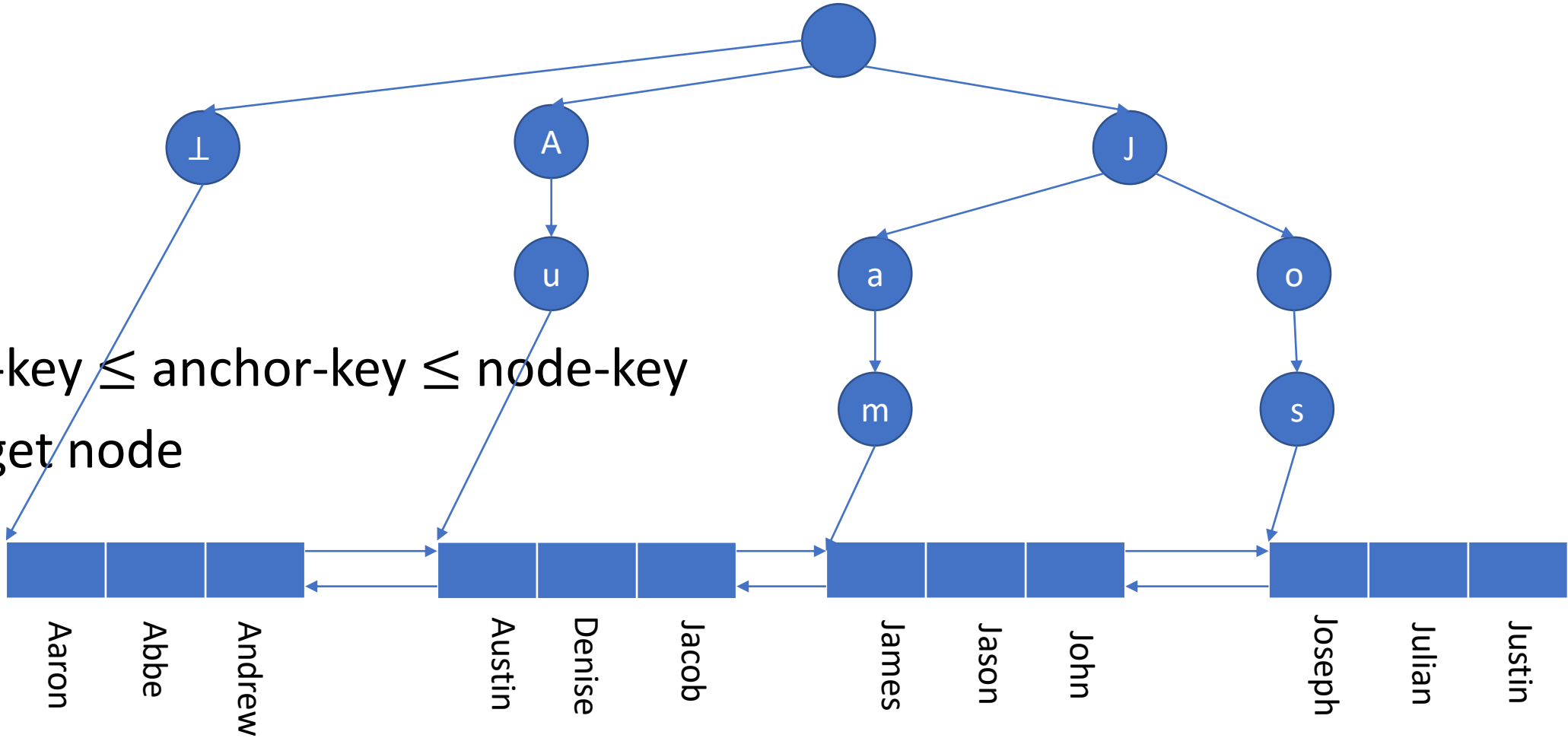


# Wormhole [1]



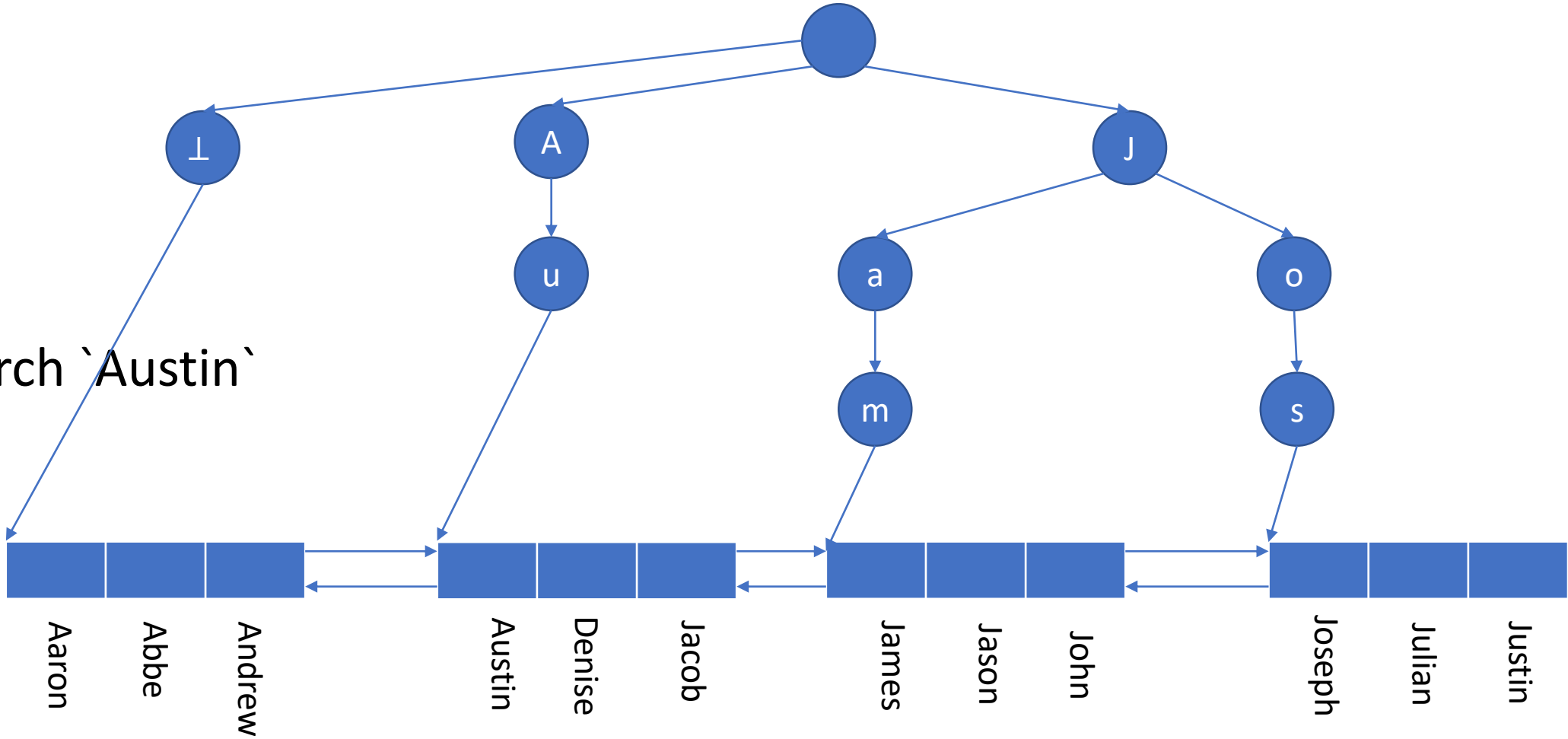
# Wormhole [1]

- left-key  $\leq$  anchor-key  $\leq$  node-key
- Target node



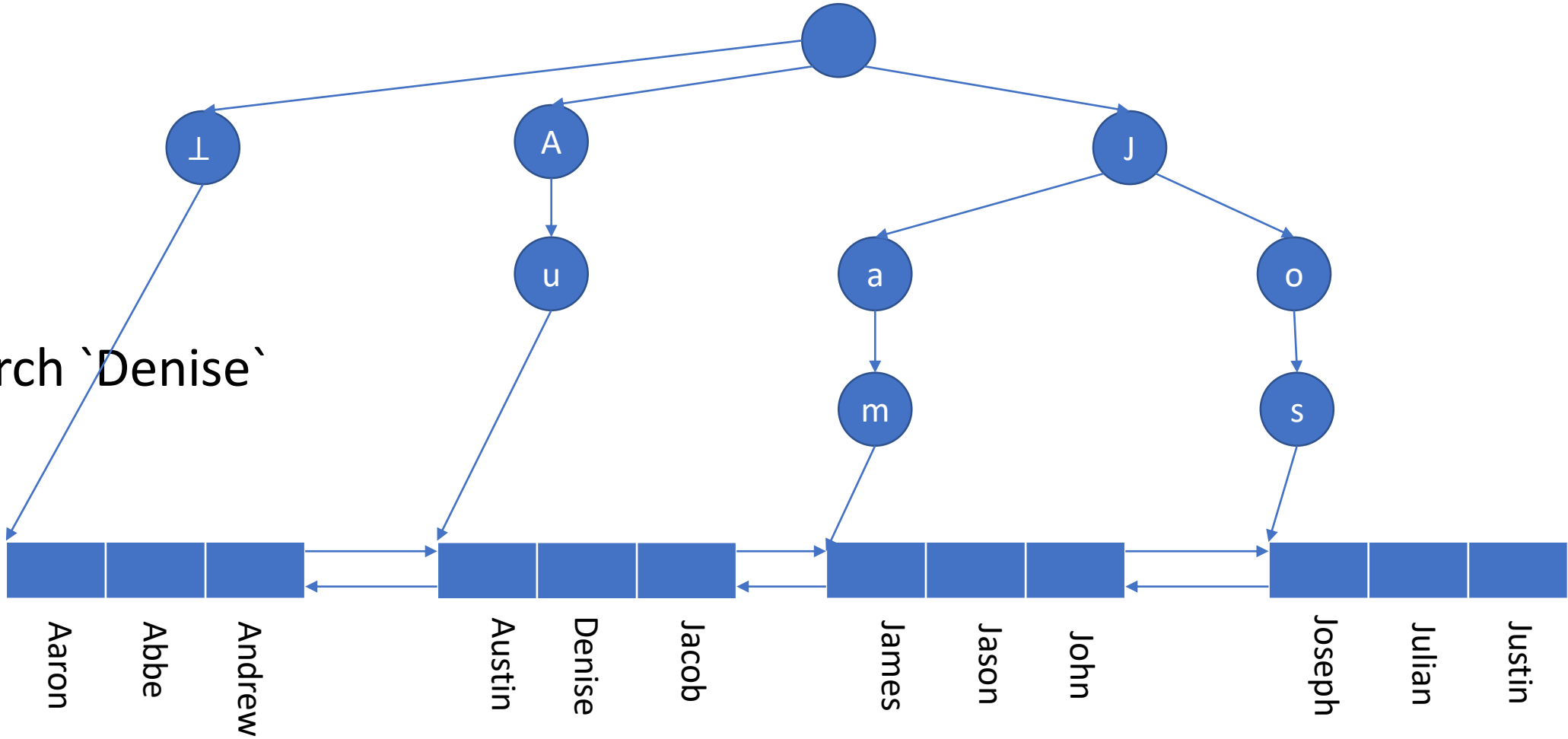
# Wormhole [1]

- Search 'Austin'



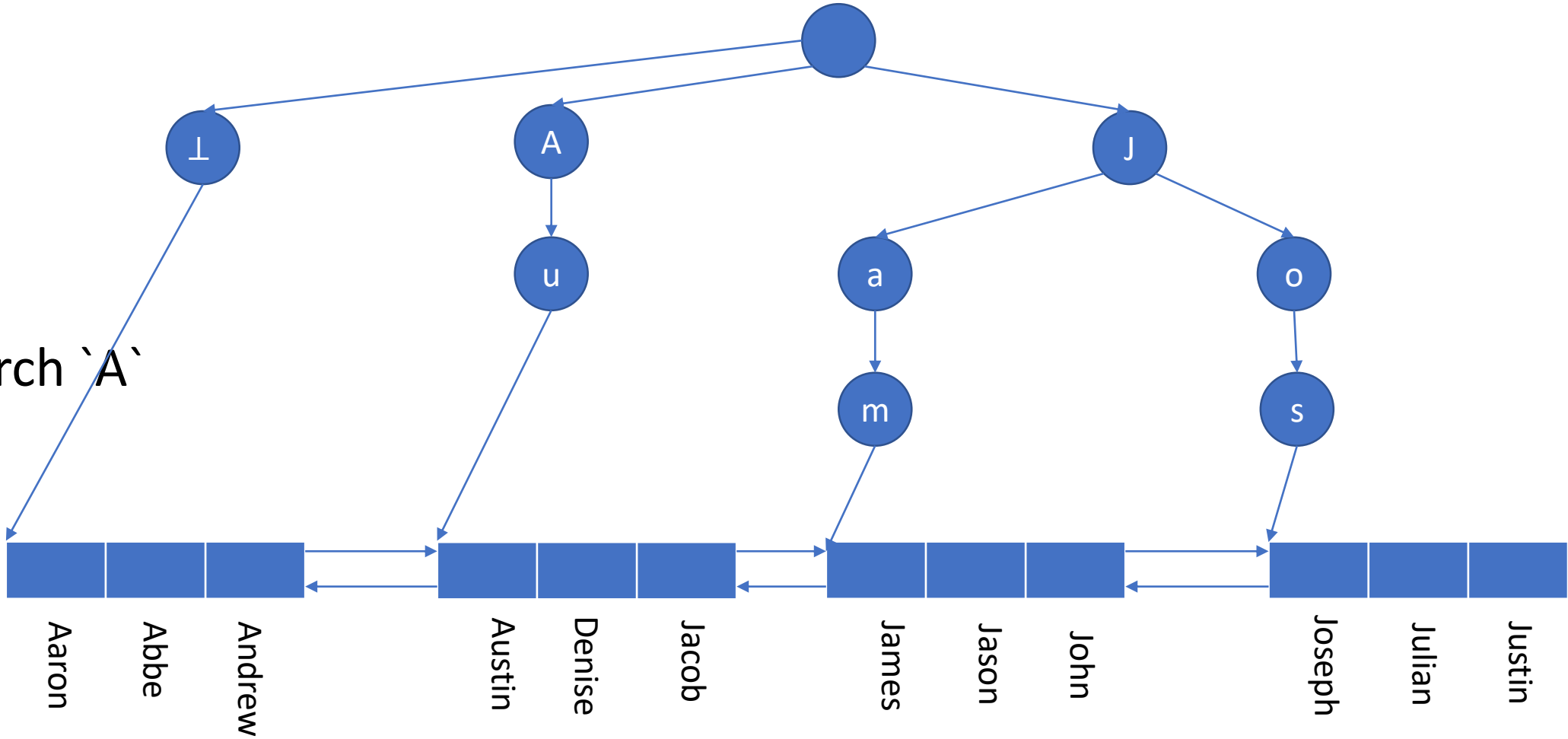
# Wormhole [1]

- Search 'Denise'



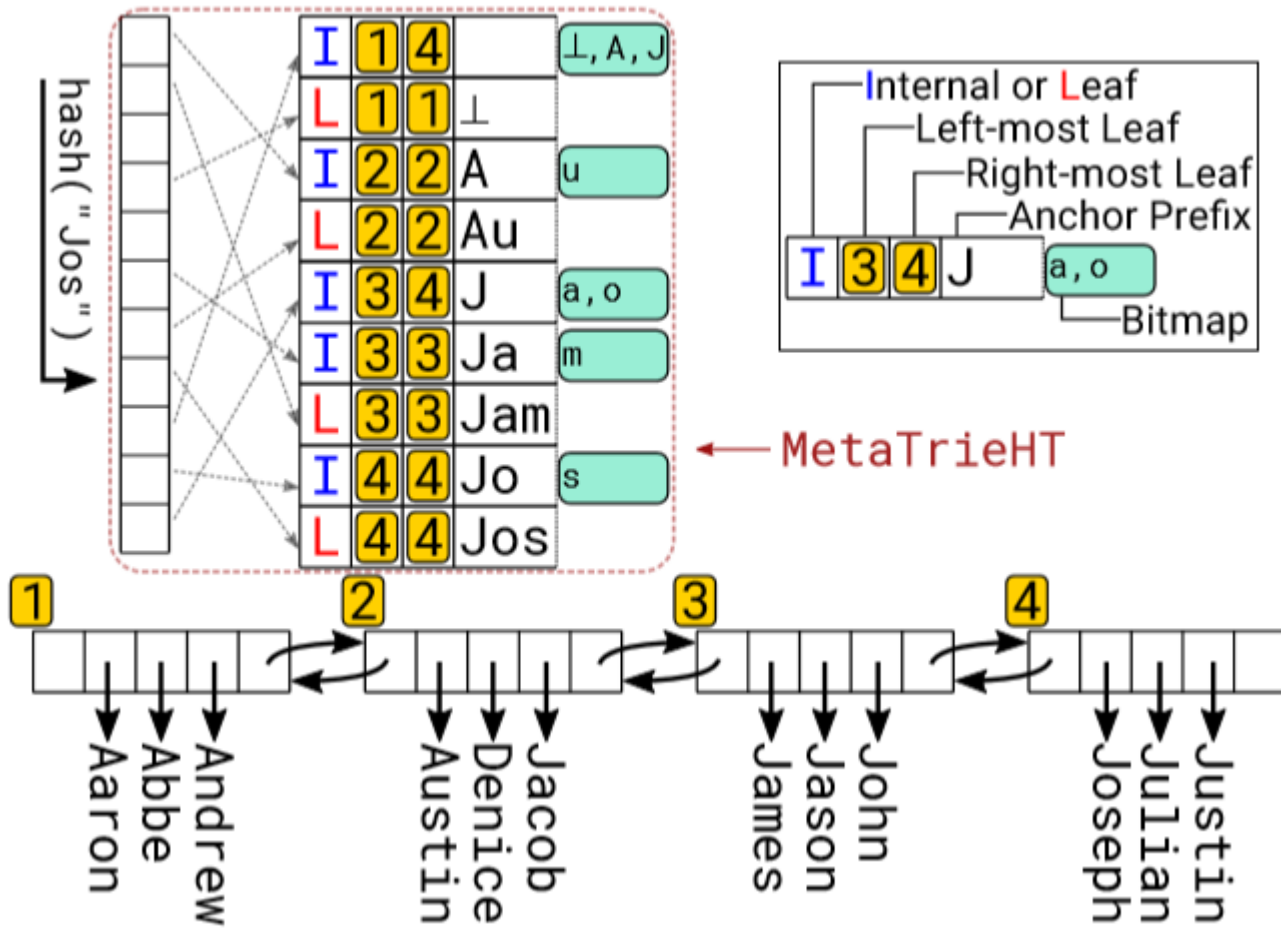
# Wormhole [1]

- Search 'A'





# Wormhole [1]

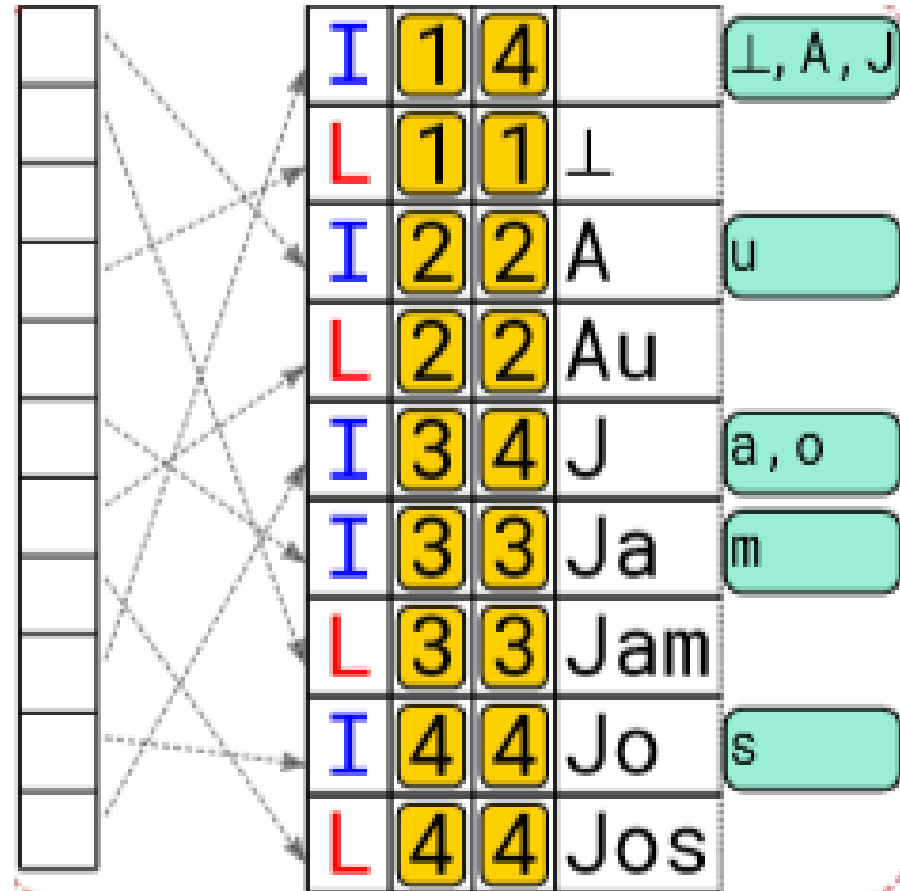


# Wormhole [1]

## Algorithm 1 Binary Search on Prefix Lengths

```
1: function searchLPM(search_key, L_anc, L_key)
2:   m ← 0;   n ← min(L_anc, L_key)+1
3:   while (m+1) < n do
4:     prefix_len ← (m+n)/2
5:     if search_key[0 : prefix_len-1] is in the trie then
6:       m ← prefix_len
7:     else n ← prefix_len
8:   return search_key[0 : m-1]
```

- search\_key = James
- L\_key = 5
- L\_anc = 4



# Wormhole [1]

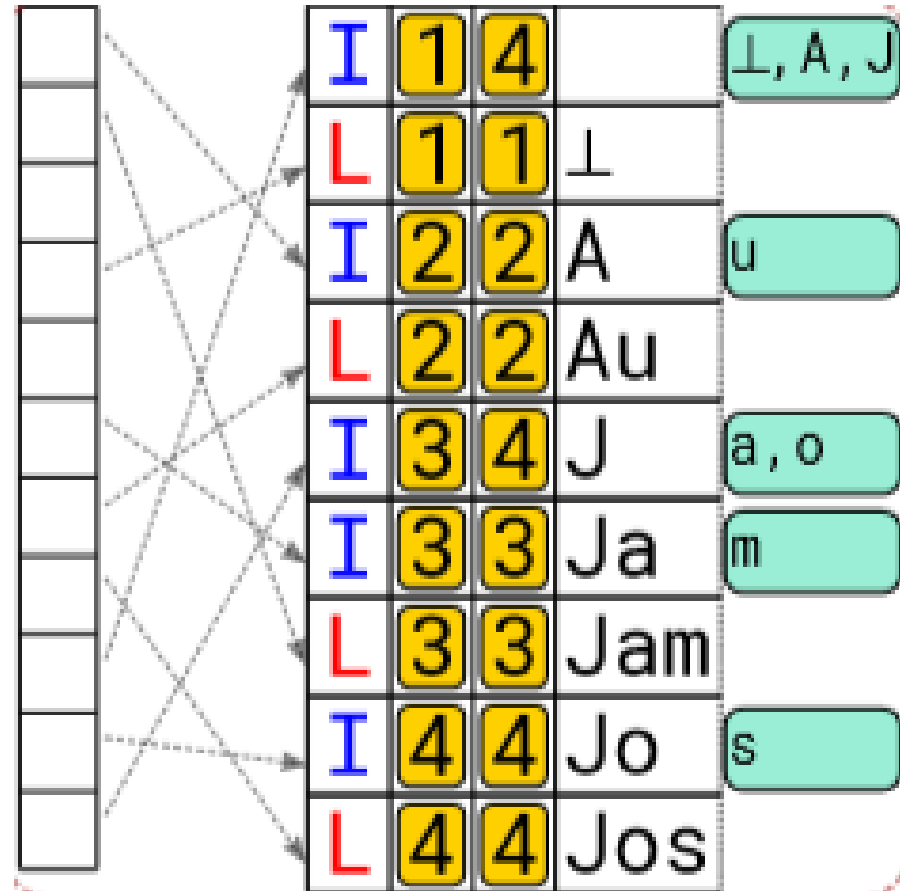
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## Algorithm 1 Binary Search on Prefix Lengths

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```

- m = 0
- n = 4

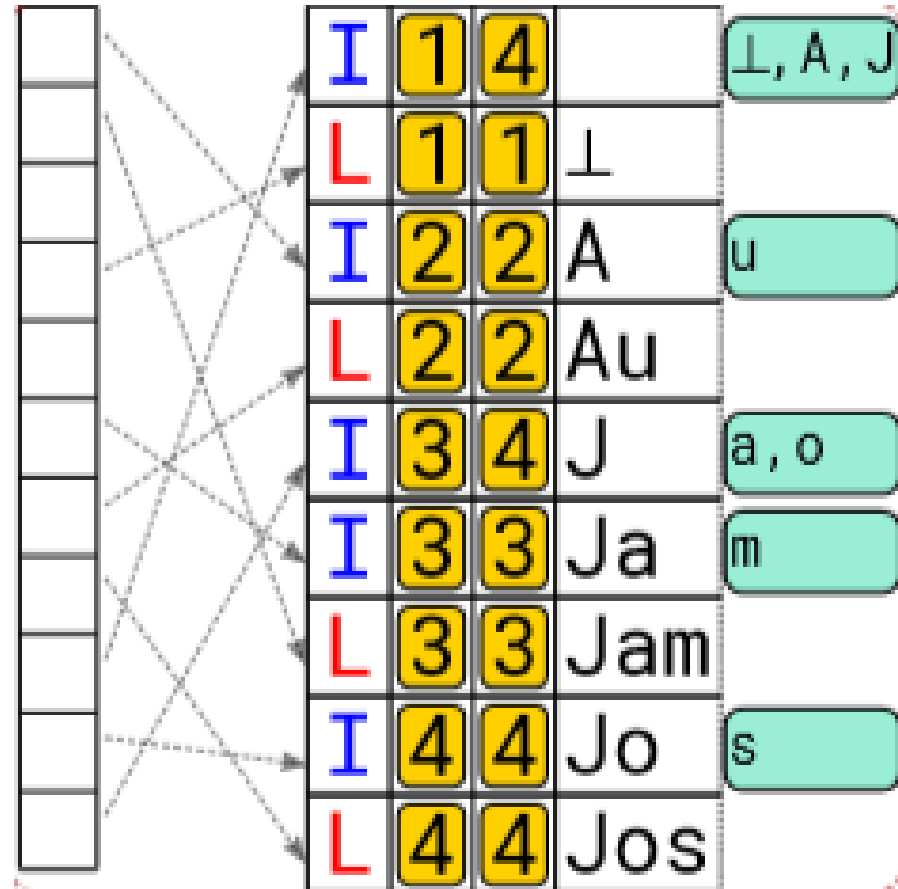


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```

- $m = 0$
- $n = 4$
- $\text{prefix\_len} = 2$

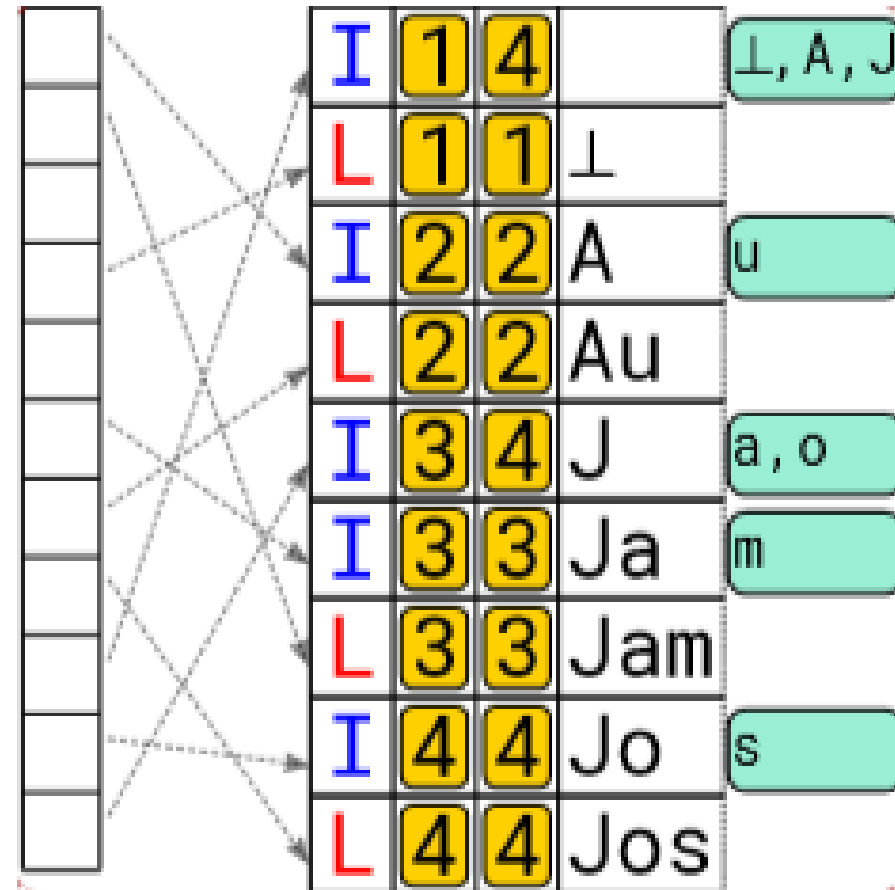


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```

- m = 2
- n = 4
- prefix\_len = 2

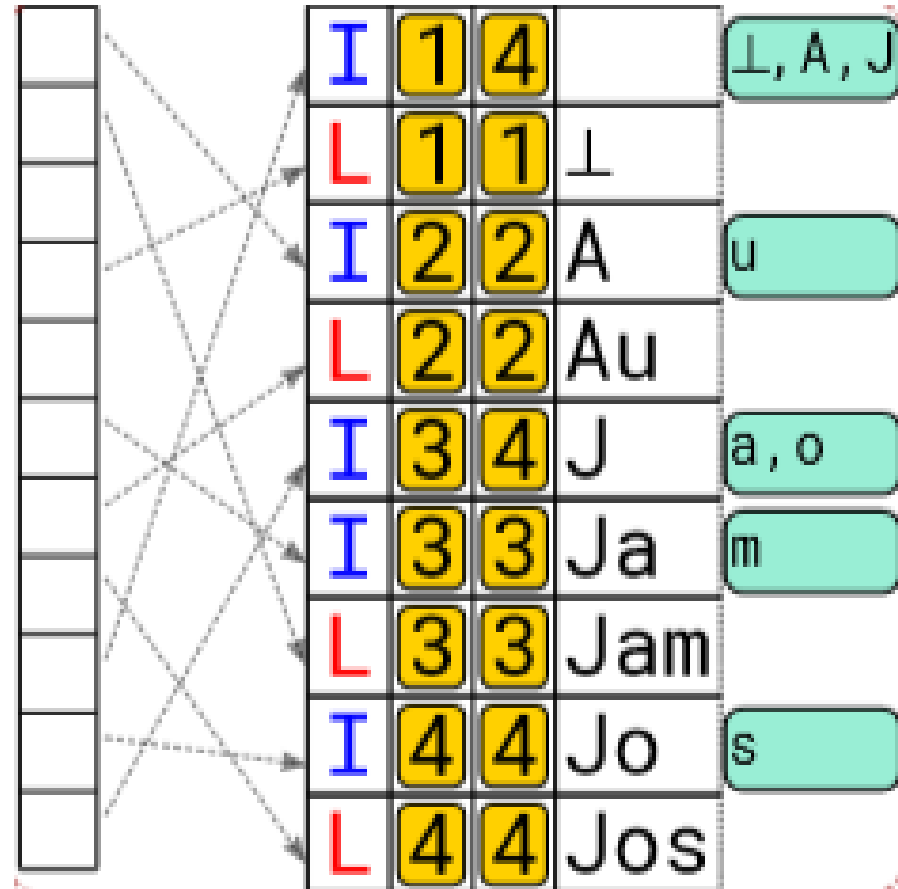


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```

- $m = 2$
- $n = 4$
- $\text{prefix\_len} = 3$



# Wormhole [1]

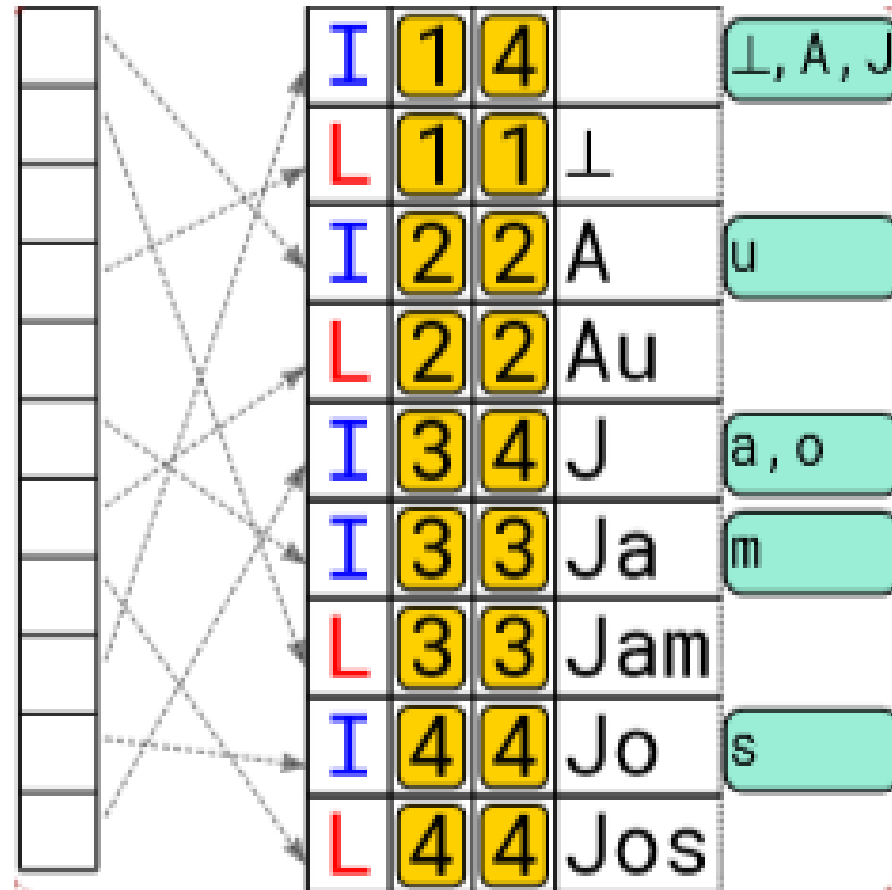
---

## Algorithm 1 Binary Search on Prefix Lengths

---

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6:       m ← prefix_len
7:     else n ← prefix_len
8:   return search_key[0 : m-1]
```

- m = 3
- n = 4
- prefix\_len = 3



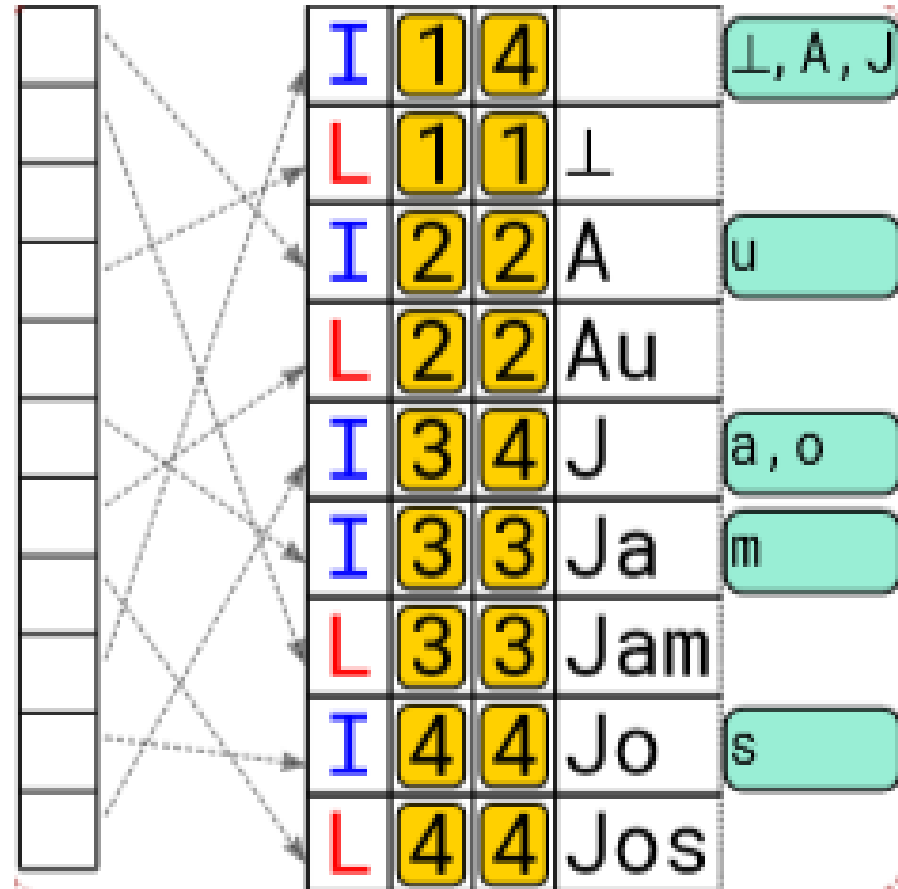
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6:       m ← prefix_len
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```

- return Jam

- $O(\log(\min(L_{anc}, L_{key})))$



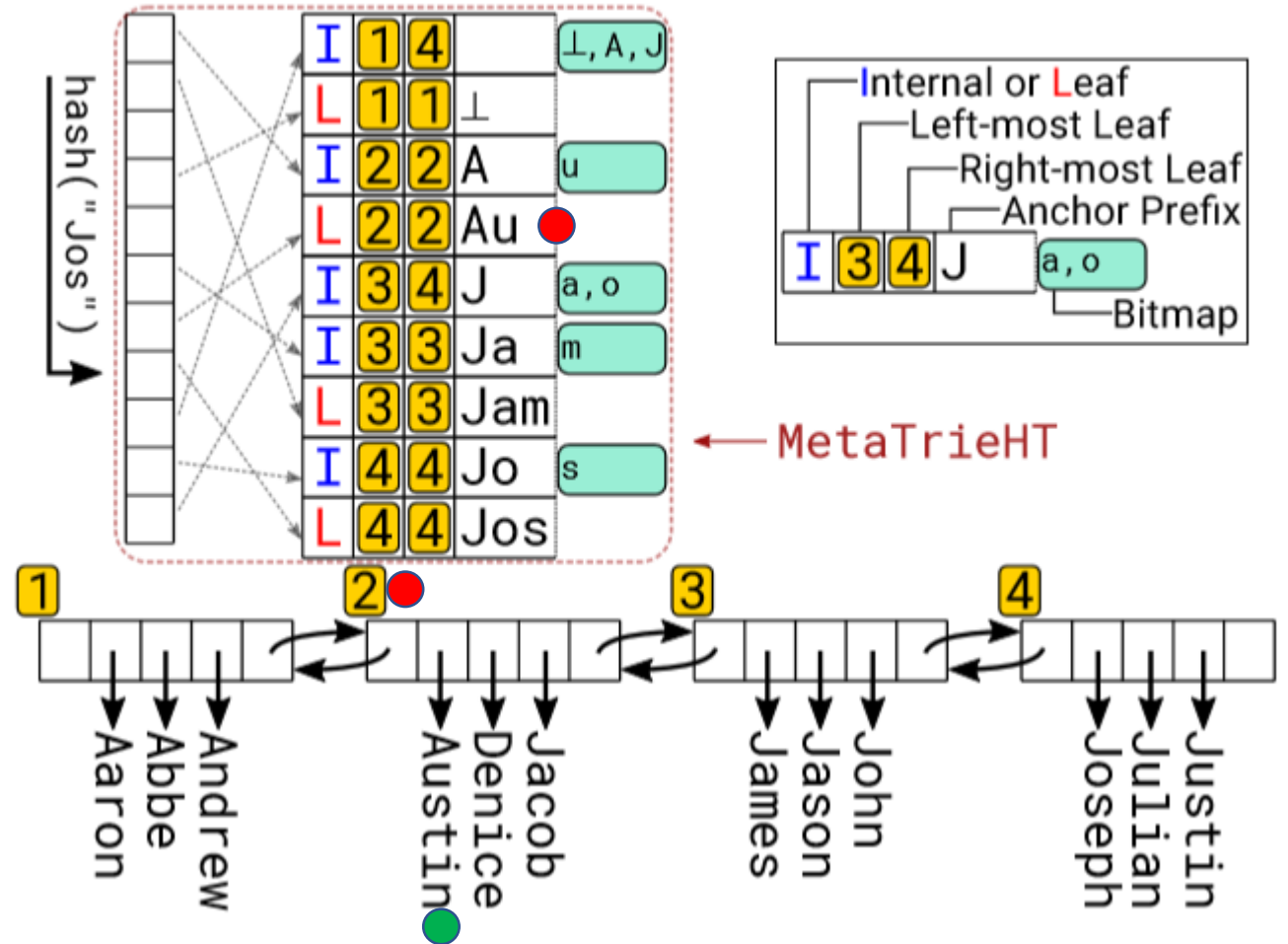


# Wormhole [1]

**function** searchTrieHT(wh, key)

```

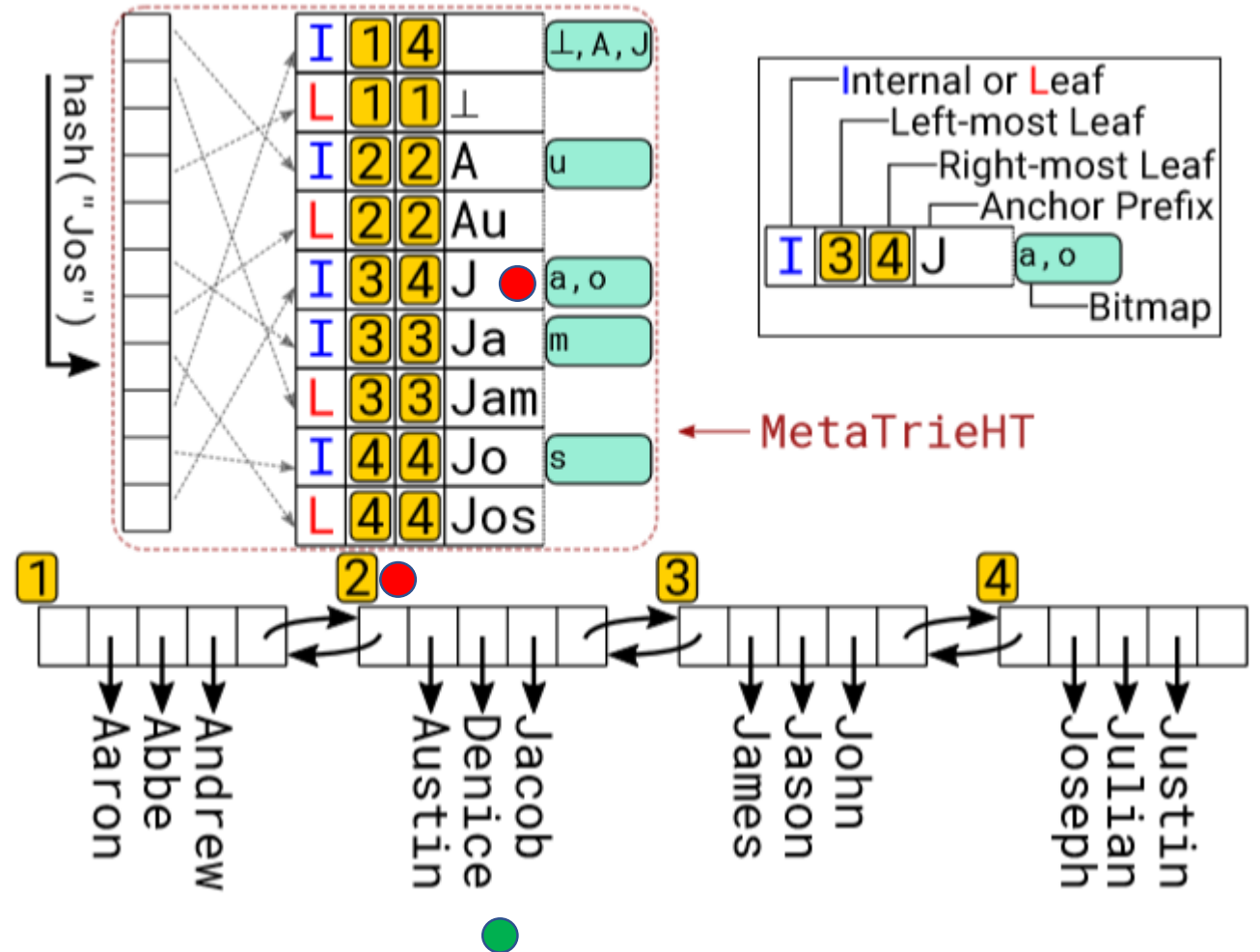
● node ← searchLPM(wh.ht, key, min(key.len, wh.maxLen))
if node.type = LEAF then return node
else if node.key.len = key.len then
  leaf ← node.leftmost
  if key < leaf.anchor then leaf ← leaf.left
  return leaf
missing ← key.tokens[node.key.len]
sibling ← findOneSibling(node.bitmap, missing)
child ← htGet(wh.ht, concat(node.key, sibling))
if child.type = LEAF then
  if sibling > missing then child ← child.left
  return child
else
  if sibling > missing then return child.leftmost.left
  else return child.rightmost
  
```



# Wormhole [1]

```

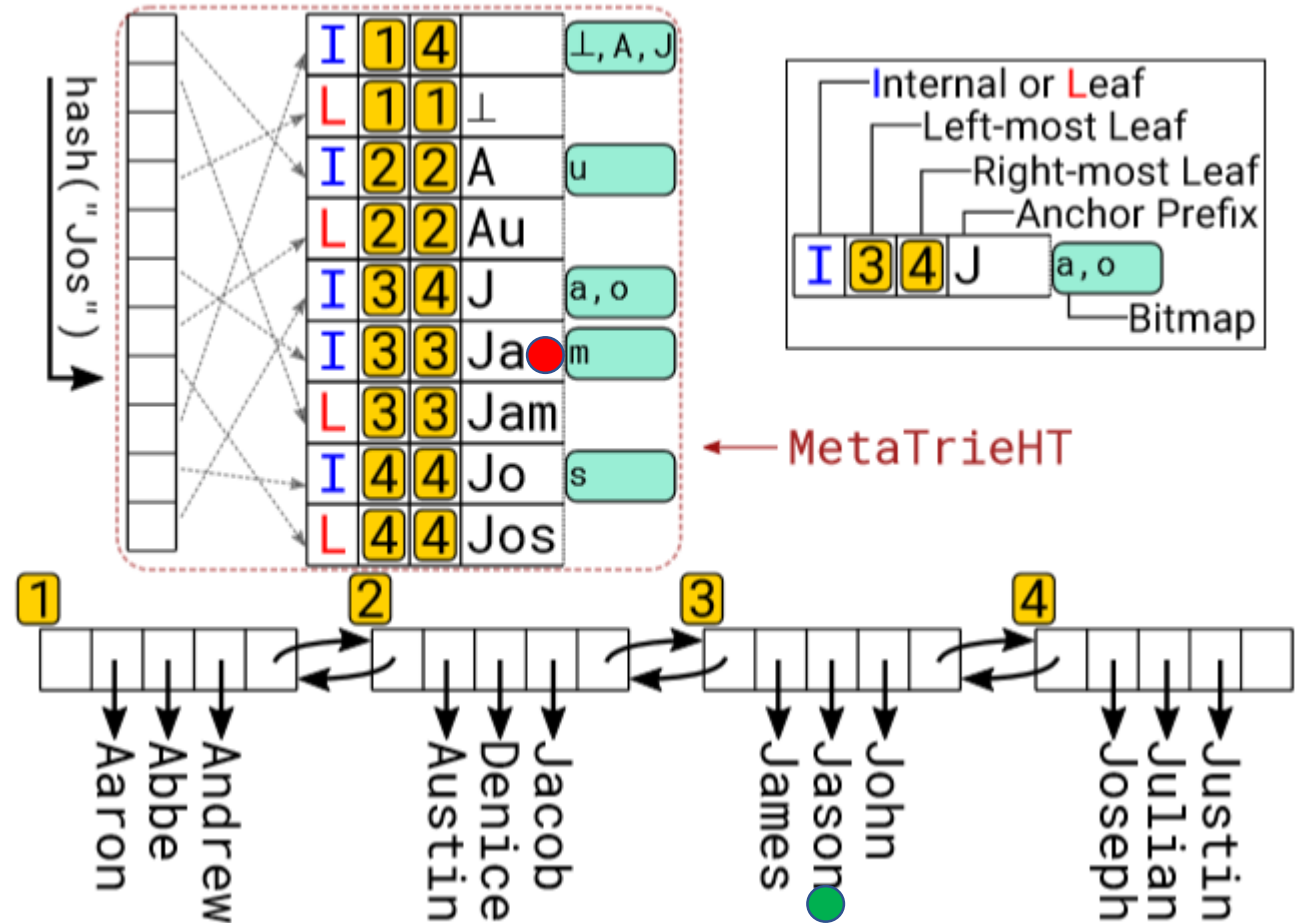
function searchTrieHT(wh, key)
  node ← searchLPM(wh.ht, key, min(key.len, wh.maxLen))
  if node.type = LEAF then return node
  if node.type = INTERNAL then
    if node.key.len = key.len then
      leaf ← node.leftmost
      if key < leaf.anchor then leaf ← leaf.left
      return leaf
    missing ← key.tokens[node.key.len]
    sibling ← findOneSibling(node.bitmap, missing)
    child ← htGet(wh.ht, concat(node.key, sibling))
    if child.type = LEAF then
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      return child
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```

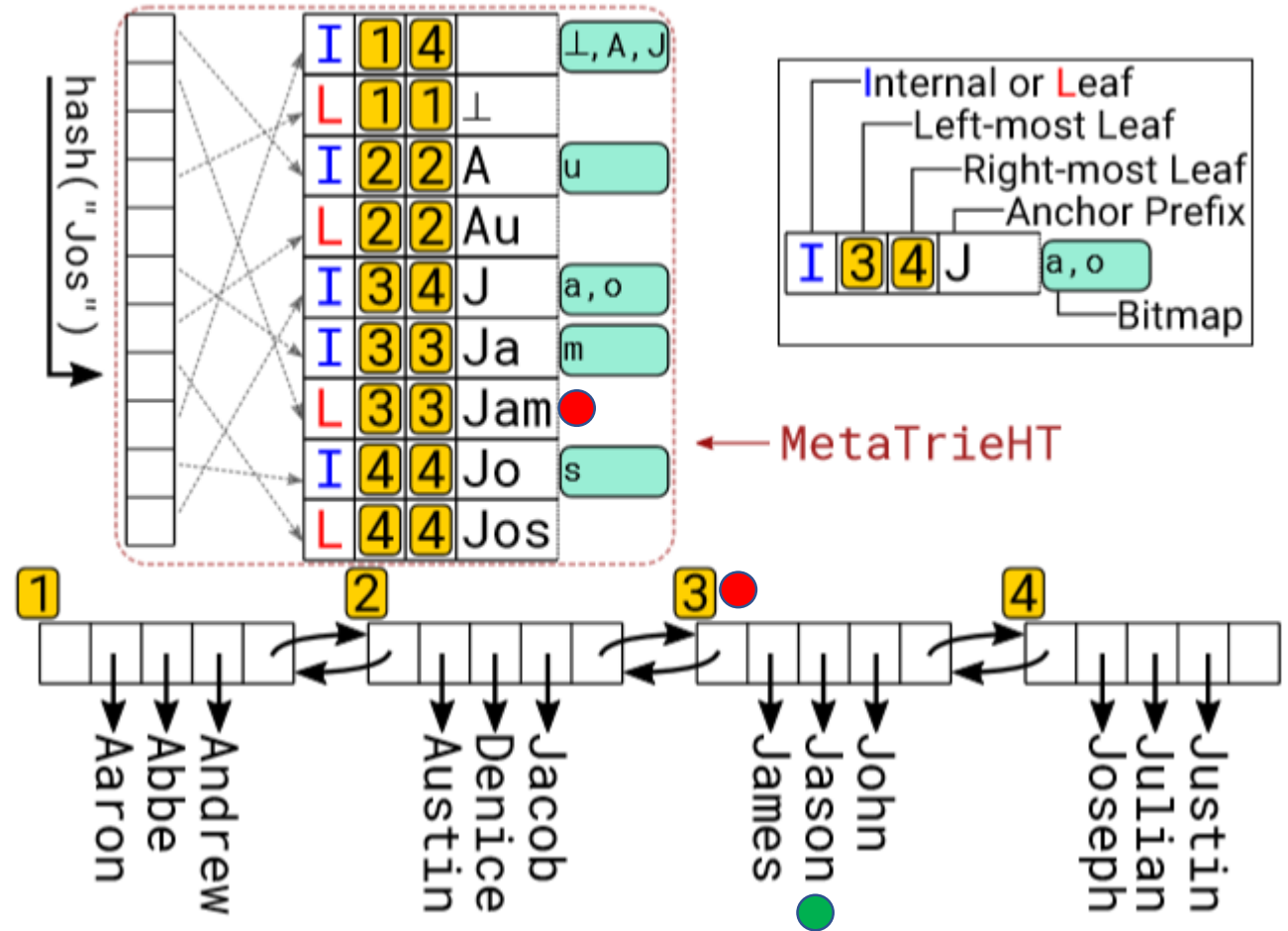
function searchTrieHT(wh, key)
  node ← searchLPM(wh.ht, key, min(key.len, wh.maxLen))
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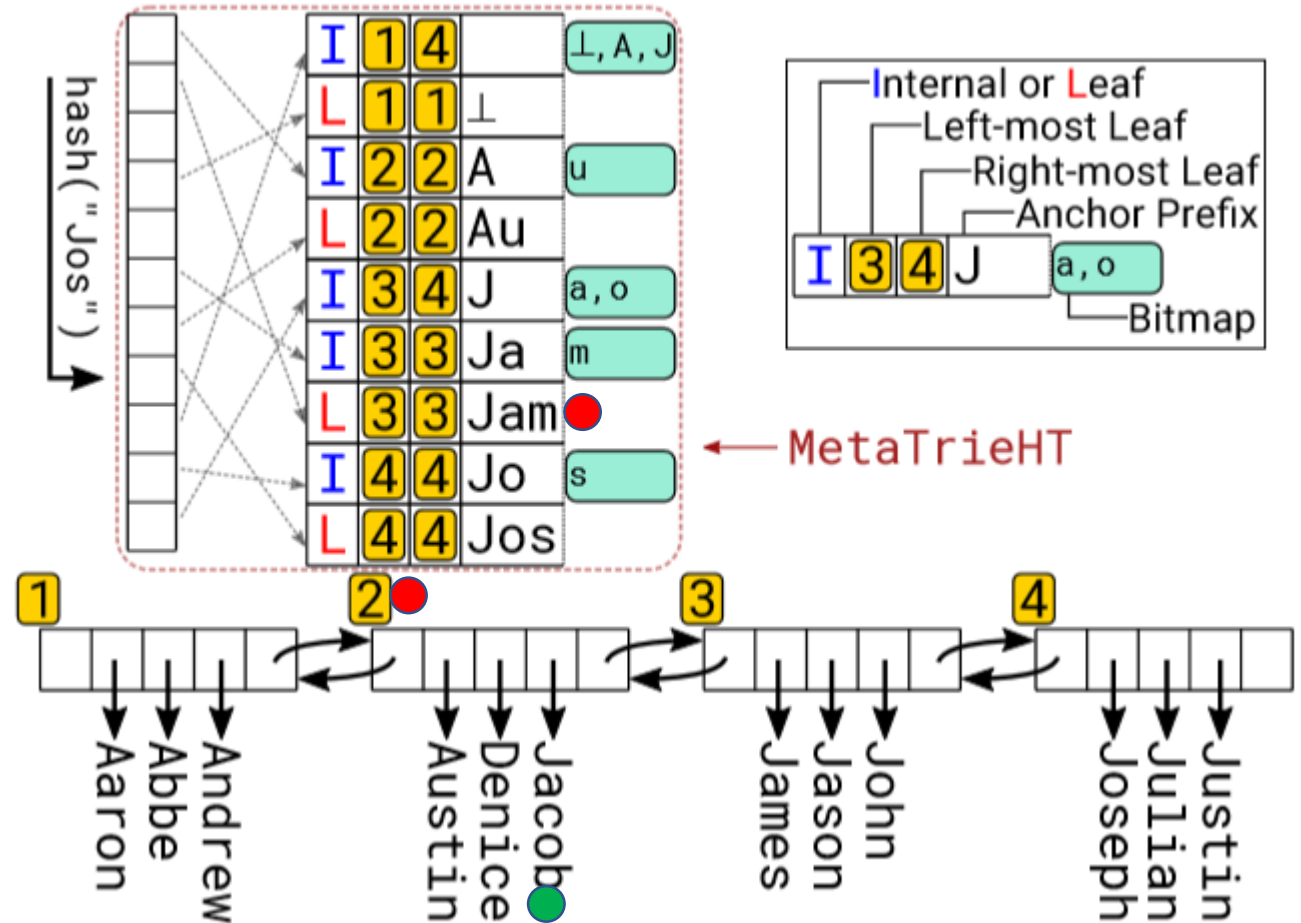
function searchTrieHT(wh, key)
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```

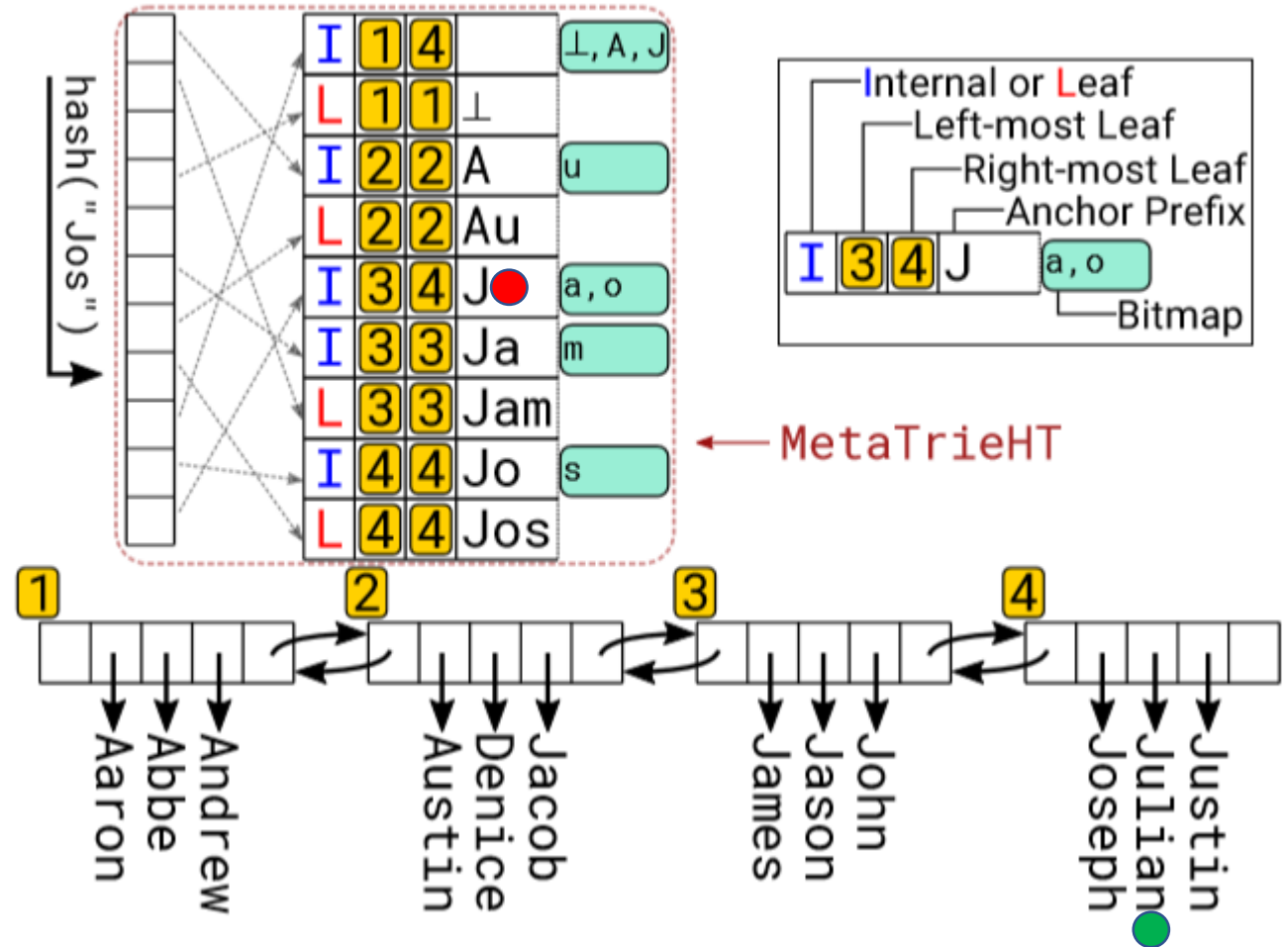
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```

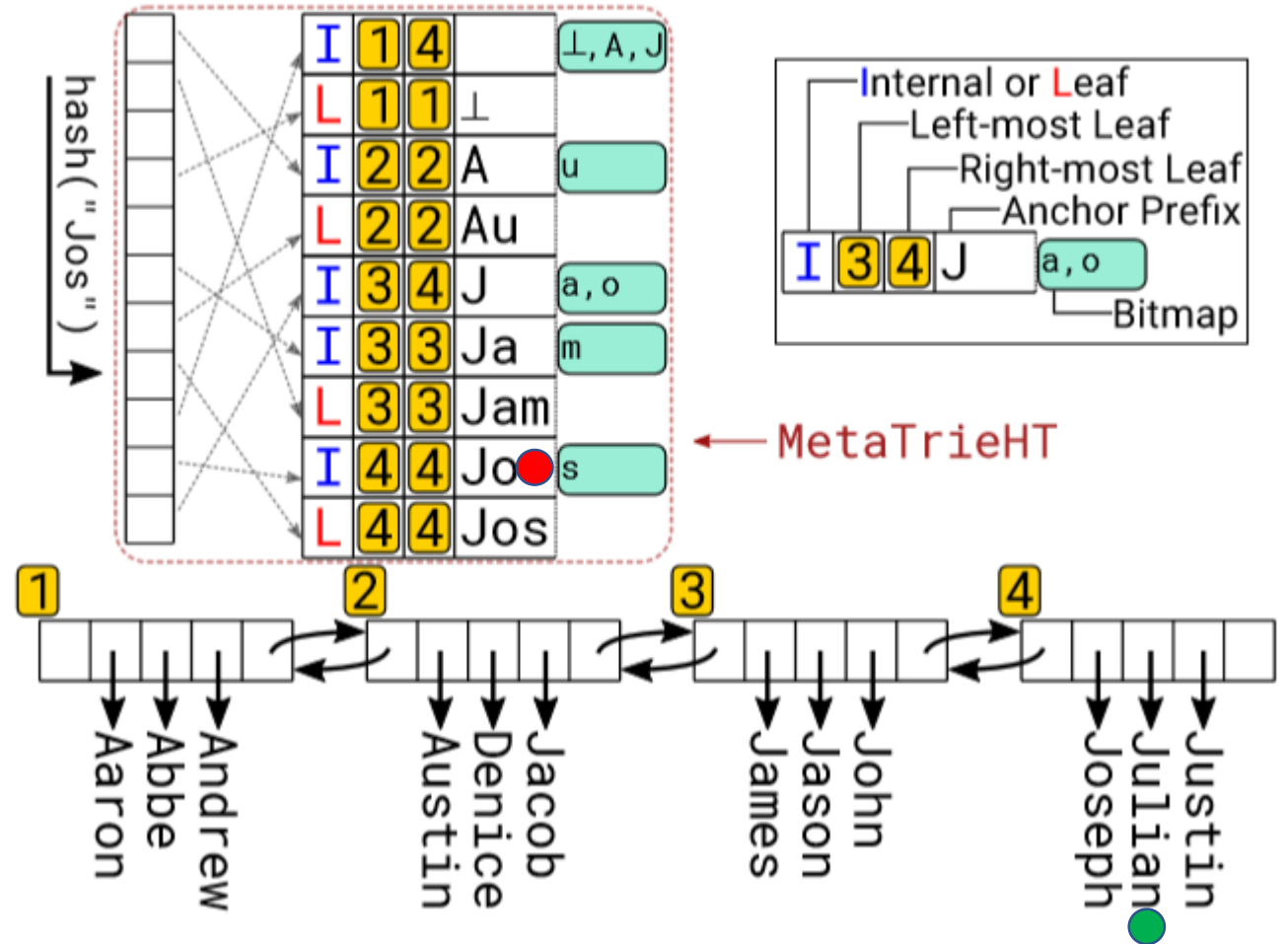
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```

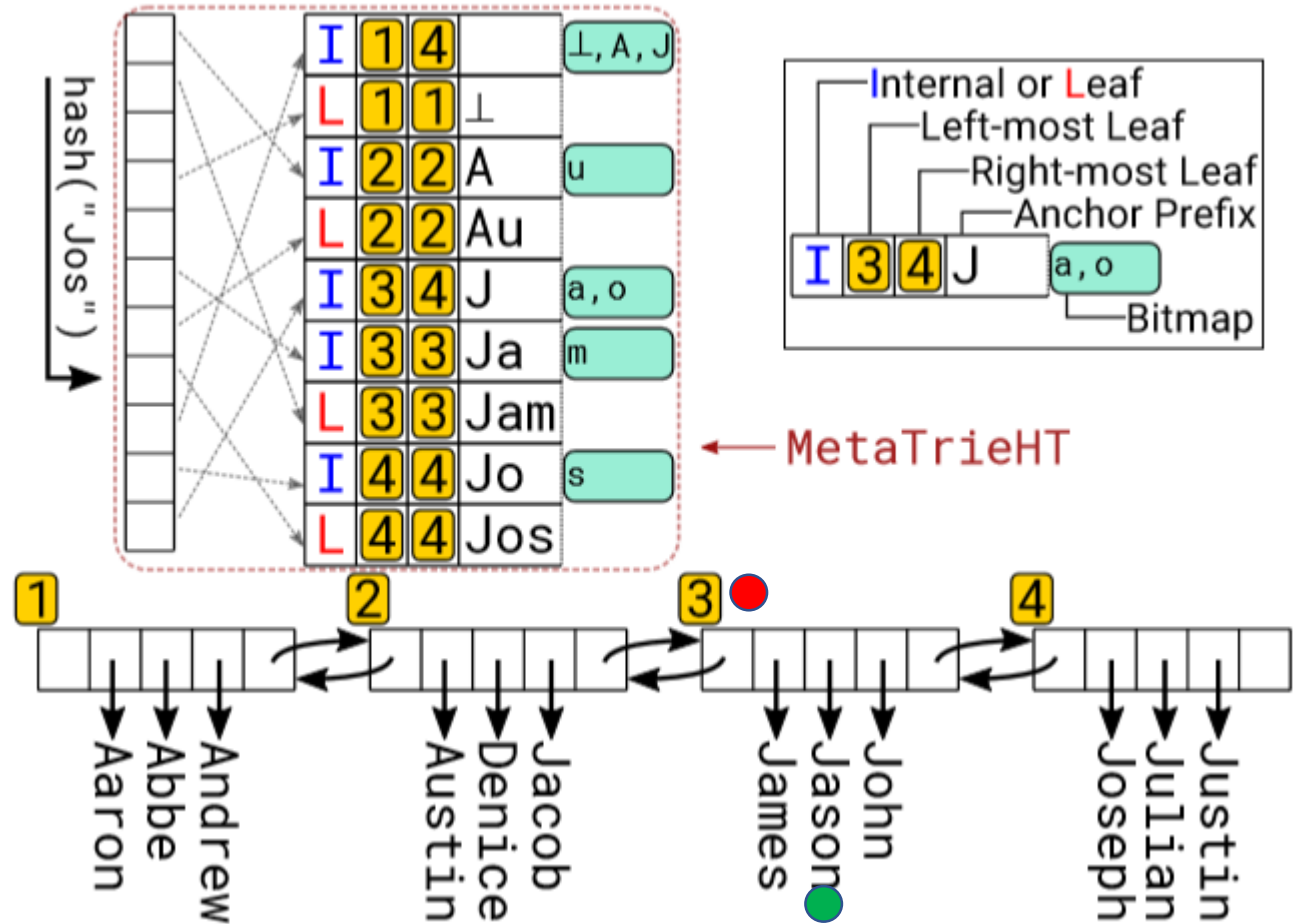
function searchTrieHT(wh, key)
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  else
    if sibling > missing then return child.leftmost.left
    else return child.rightmost
  
```



# Wormhole [1]

```

function GET(wh, key)
  leaf ← searchTrieHT(wh, key);    i ← pointSearchLeaf(leaf, key)
  if (i < leaf.size) and (key = leaf.hashArray[i].key) then
    return leaf.hashArray[i]
  else return NULL
  
```

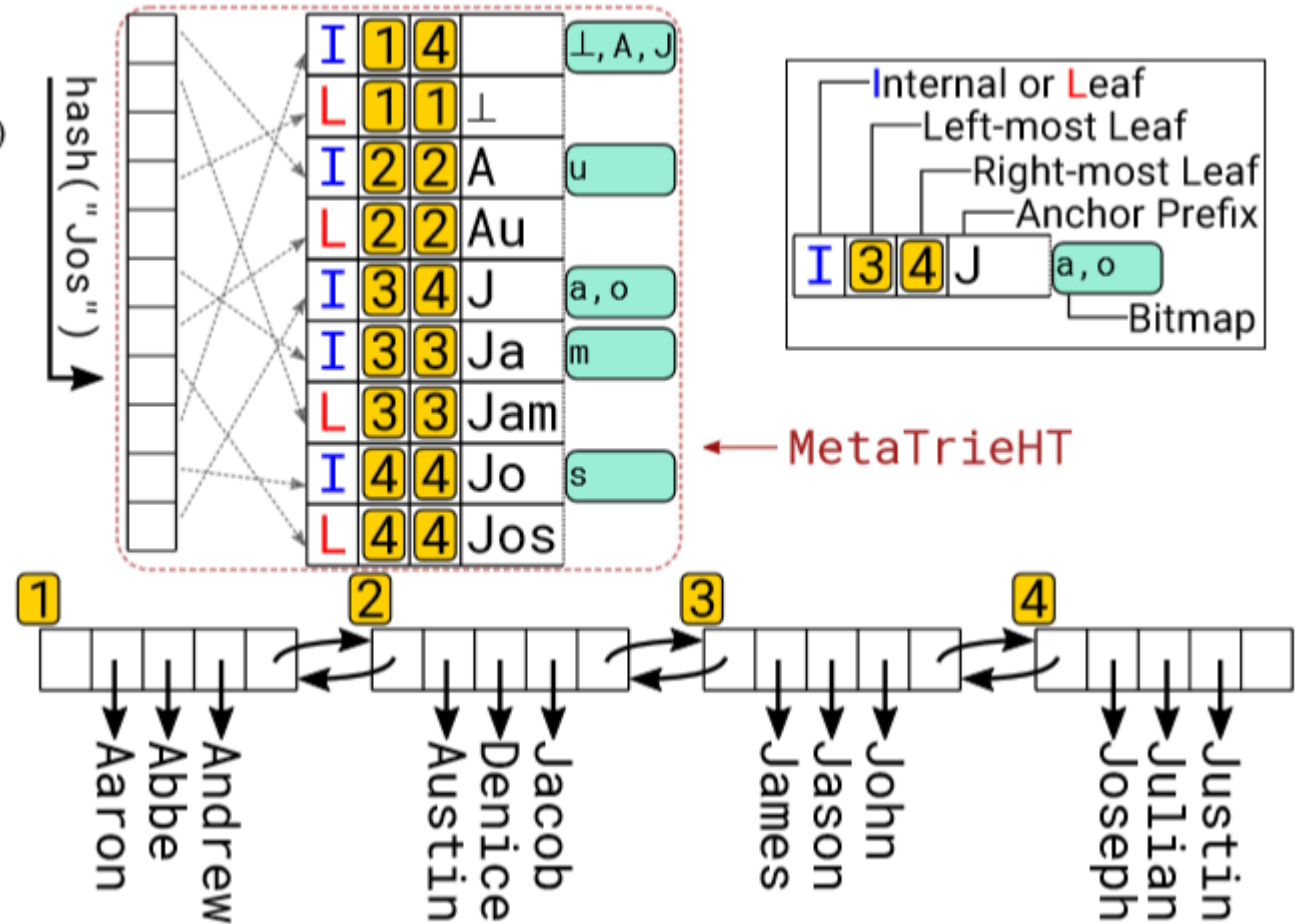




# Wormhole [1]

```

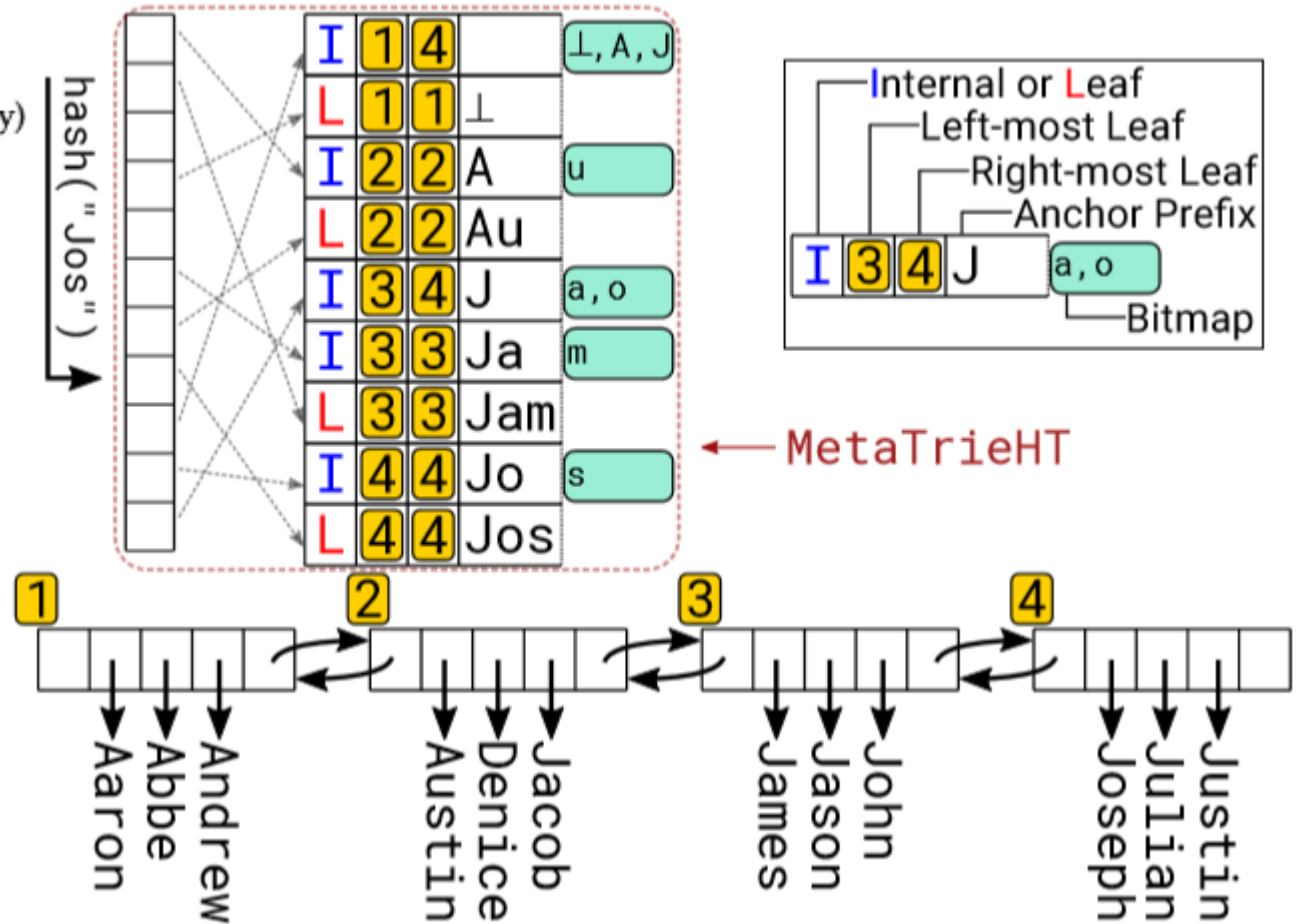
function DEL(wh, key)
  leaf ← searchTrieHT(wh, key);    i ← pointSearchLeaf(leaf, key)
  if (i < leaf.size) and (key = leaf.hashArray[i].key) then
    leafDelete(leaf, i)
    if (leaf.size + leaf.left.size) < MergeSize then
      merge(wh, leaf.left, leaf)
    else if (leaf.size + leaf.right.size) < MergeSize then
      merge(wh, leaf, leaf.right)
  
```



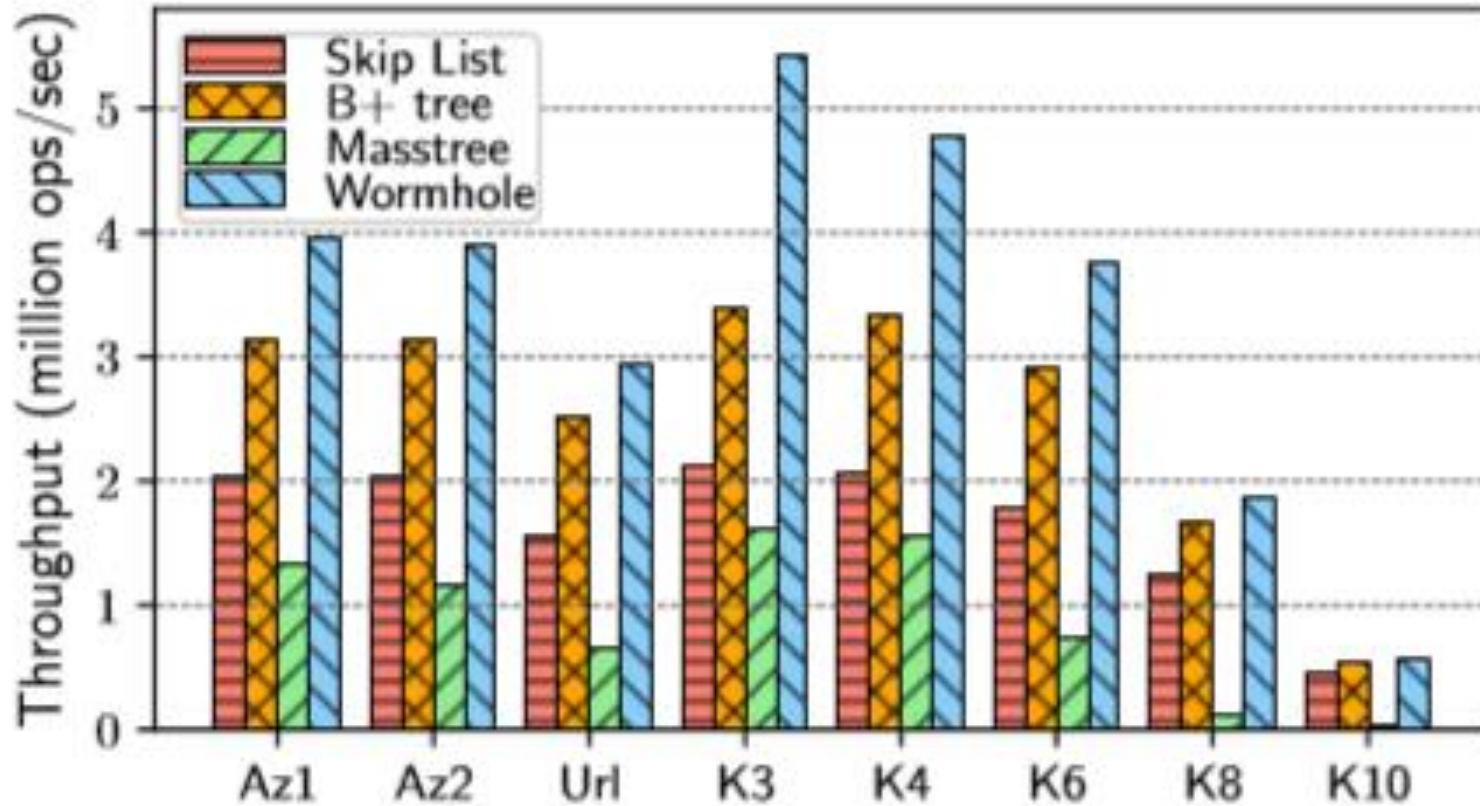
# Wormhole [1]

```

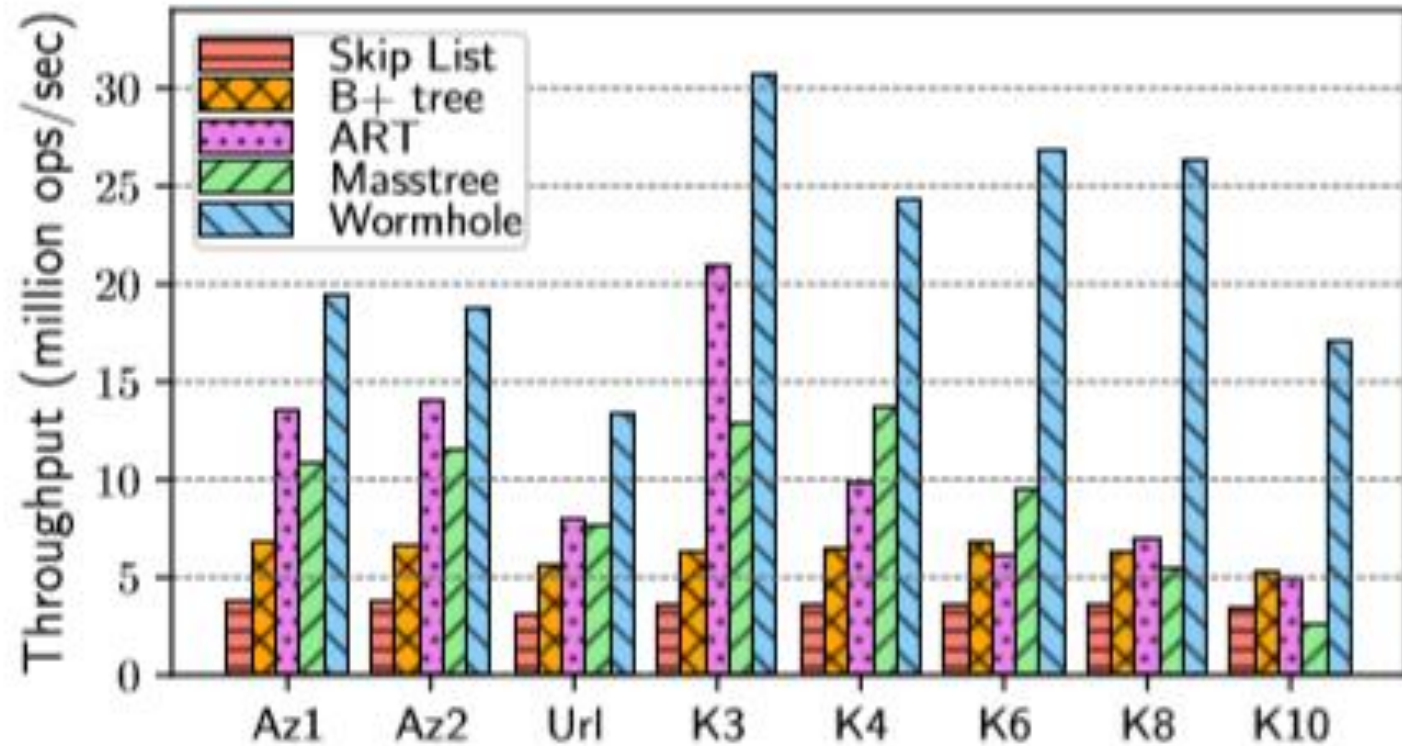
function SET(wh, key, value)
  leaf ← searchTrieHT(wh, key);   i ← pointSearchLeaf(leaf, key)
  if (i < leaf.size) and (key = leaf.hashArray[i].key) then
    leaf.hashArray[i].value ← value
  else
    if leaf.size = MaxLeafSize then
      left, right ← split(wh, leaf)
      if key < right.anchor then
        leaf ← left
      else leaf ← right
    leafInsert(leaf, key, value)
  
```



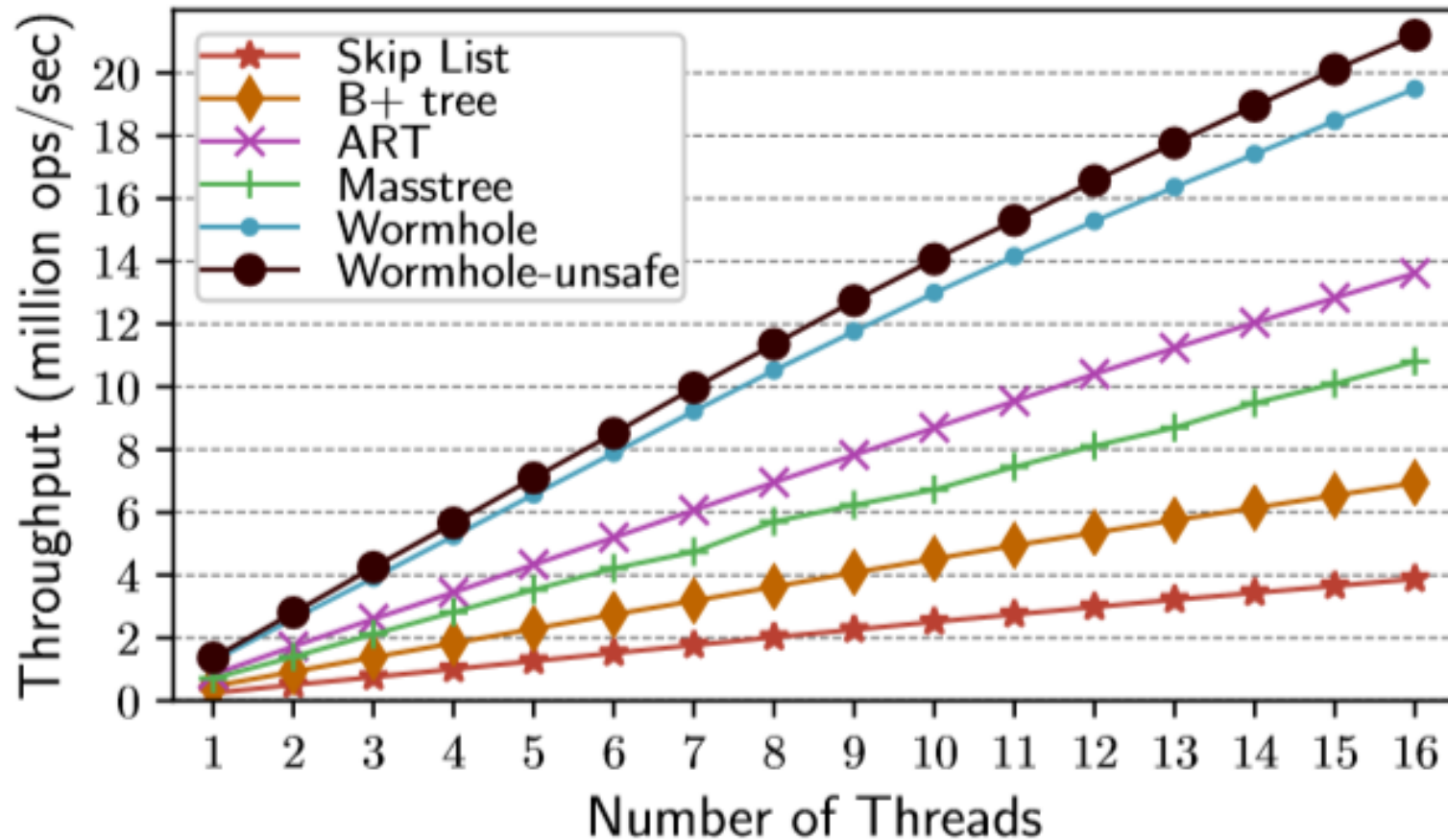
# Throughput of range lookups [1]



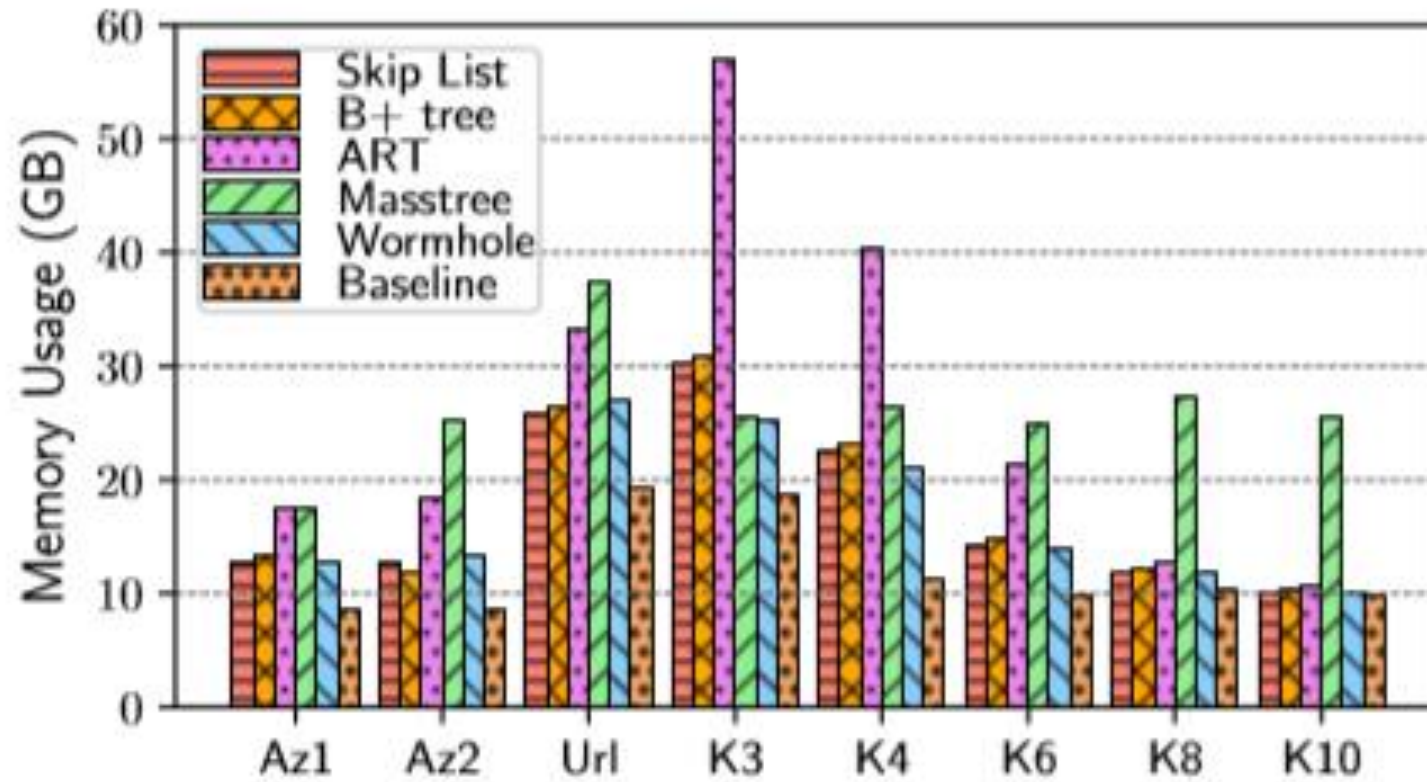
# Lookup throughput on local CPU [1]



# Lookup throughput with threads [1]



# Memory usage of the indexes [1]



# Zusammenfassung

	GET	SET	DEL	GET_RANGE
Hashtabellen	$O(1)$	$O(1)$	$O(1)$	nein
B+Baum	$O(\log N)$	$O(\log N)$	$O(\log N)$	$O(\log N + k)$
Präfix Baum	$O(L)$	$O(L)$	$O(L)$	ja
Wormhole	$O(\log L)$	$O(\log L)$	$O(\log L)$	$O(\log L + k)$

# Zusammenfassung

- Aus B+, Trie und Hash Table neue Index Struktur zsm gebaut
- GET, SET, DEL(, range search) in  $O(\log L)$
- Platzkomplexität vergleichbar mit B+ Baum
- Niedrige Zeitkomplexität



# Quellen

1. Paper: Wormhole: A Fast Ordered Index for In-memory Data Management
2. AD Folien WS 18/19
3. GDB Folien WS 18/19
4. <https://www.geeksforgeeks.org/trie-insert-and-search/>
5. <https://www.geeksforgeeks.org/advantages-trie-data-structure/>